Chapter 4

Heat Transfer in Food Processing

The most common processes found in a food processing plant involve heating and cooling of foods. In the modern industrialized food industry, we commonly find unit operations such as refrigeration, freezing, thermal sterilization, drying, and evaporation. These unit operations involve the transfer of heat between a product and some heating or cooling medium. Heating and cooling of food products is necessary to prevent microbial and enzymatic degradation. In addition, desired sensorial properties—color, flavor, texture—are imparted to foods when they are heated or cooled.

The study of heat transfer is important because it provides a basis for understanding how various food processes operate. In this chapter, we will study the fundamentals of heat transfer and learn how they are related to the design and operation of food processing equipment.

We will begin by studying heat-exchange equipment. We will observe that there is a wide variety of heat-exchange equipment available for food applications. This description will identify the need to study properties of foods that affect the design and operation of heat exchangers. Thereafter, we will examine various approaches to obtaining thermal properties of foods. We will consider basic modes of heat transfer such as conduction, convection, and radiation. Simple mathematical equations will be developed to allow prediction of heat transfer in solid as well as liquid foods. These mathematical equations will provide us with sufficient tools to design and evaluate the performance of simple heat exchangers. Next, we will consider more complicated situations arising from heat transfer under unsteady-state conditions, when temperature changes with time. A good understanding of the various

All icons in this chapter refer to the author's web site, which is independently owned and operated. Academic Press is not responsible for the content or operation of the author's web site. Please direct your web site comments and questions to the
author: Professor R. Paul Singh, Department of Biological and Agricultural Engineering, University of California, Davis, CA 95616, USA.
Email: rps@rpaulsingh.com

concepts presented in this chapter is important, since they will be the basis for topics in the following chapters.

4.1 SYSTEMS FOR HEATING AND COOLING FOOD PRODUCTS

In a food processing plant, heating and cooling of foods is conducted in equipment called heat exchangers. As shown in Figure 4.1, heat exchangers can be broadly classified into non-contact and contact types. As the name implies, in non-contact-type heat exchangers, the product and heating or cooling medium are kept physically separated, usually by a thin wall. On the other hand, in contact-type heat exchangers, there is direct physical contact between the product and the heating or cooling streams.

For example, in a steam-injection system, steam is directly injected into the product to be heated. In a plate heat exchanger, a thin metal plate separates the product stream from the heating or cooling stream while allowing heat transfer to take place without mixing. We will discuss some of the commonly used heat exchangers in the food industry in the following subsections.

4.1.1 Plate Heat Exchanger

The plate heat exchanger invented more than 70 years ago has found wide application in the dairy and food beverage industry. A schematic of a plate heat exchanger is shown in Figure 4.2. This heat exchanger consists of a series of parallel, closely spaced stainless-steel

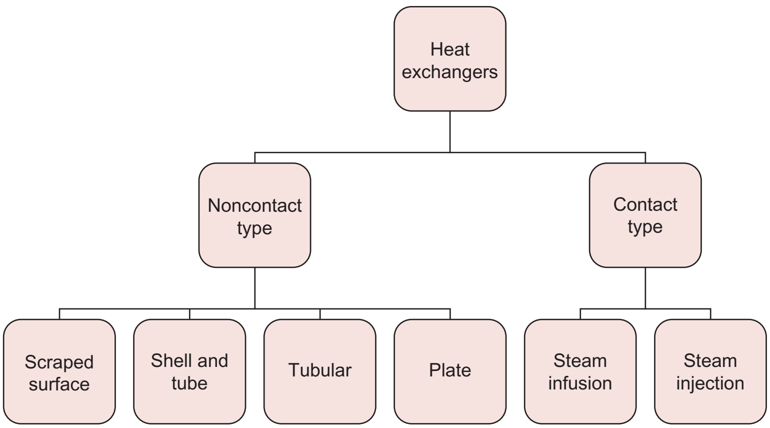
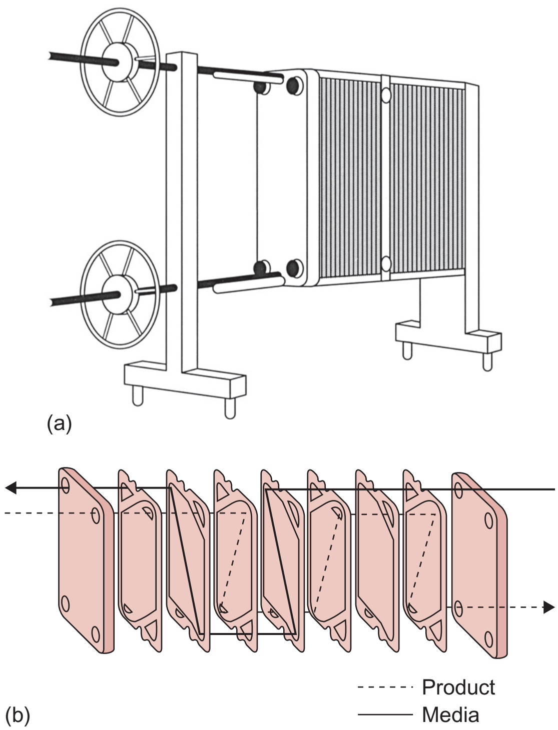


Figure 4.1 Classification of commonly used heat exchangers.

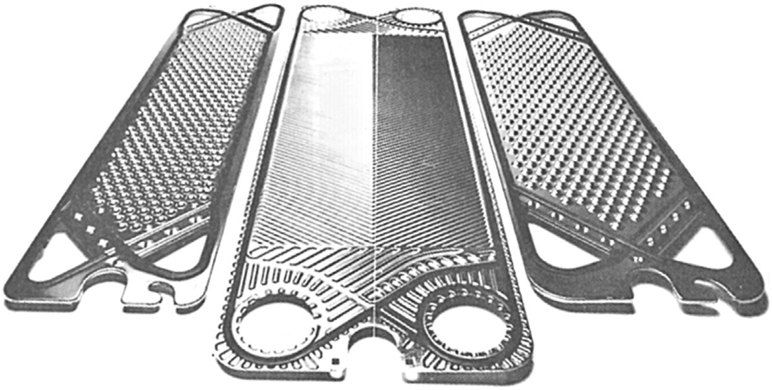


■ **Figure 4.2** (a) Plate heat exchanger. (b) Schematic view of fluid flow between plates. (Courtesy of Cherry-Burrell Corporation)

plates pressed in a frame. Gaskets, made of natural or synthetic rubber, seal the plate edges and ports to prevent intermixing of liquids. These gaskets help to direct the heating or cooling and the product streams into the respective alternate gaps. The direction of the product stream versus the heating/cooling stream can be either parallel flow (same direction) or counterflow (opposite direction) to each other. We will discuss the influence of flow direction on the performance of the heat exchanger later in Section 4.4.7.

The plates used in the plate heat exchanger are constructed from stainless steel: Special patterns are pressed on the plates to cause increased turbulence in the product stream, thus achieving better heat transfer. An example of such a pattern is a shallow herringbone-ribbed design, as shown in Figure 4.3.

Plate heat exchangers are suitable for low-viscosity (<5 Pa s) liquid foods. If suspended solids are present, the equivalent diameter of the



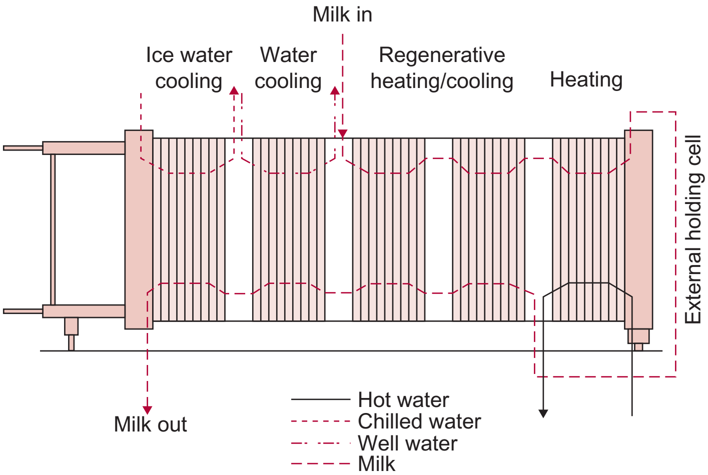
■ **Figure 4.3** Patterns pressed on plates used on a plate heat exchanger. (Courtesy of Cherry-Burrell Corporation)

particulates should be less than 0.3 cm. Larger particulates can bridge across the plate contact points and "burn on" in the heating section.

In industrial-size plate heat exchangers, product flow rates from 5000 to 20,000 kg/h often are obtained. When using plate heat exchangers, care should be taken to minimize the deposition of solid food material such as milk proteins on the surface of the plates. This deposition, also called fouling, will decrease the heat transfer rate from the heating medium to the product; in addition, the pressure drop will increase over a period of time. Eventually, the process is stopped and the plates are cleaned. For dairy products, which require ultra-high-temperature applications, the process time is often limited to 3–4 h. Plate heat exchangers offer the following advantages:

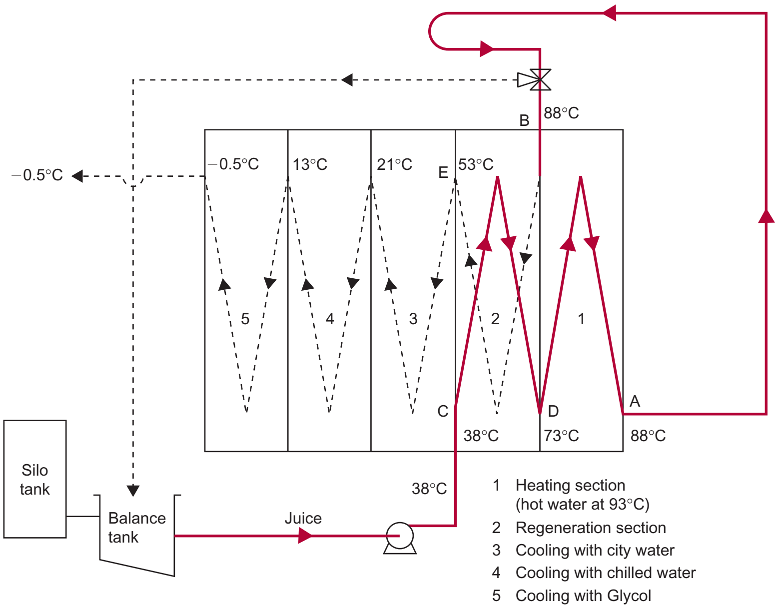
- The maintenance of these heat exchangers is simple, and they can be easily and quickly dismantled for product surface inspection.
- The plate heat exchangers have a sanitary design for food applications.
- Their capacity can easily be increased by adding more plates to the frame.
- With plate heat exchangers, we can heat or cool product to within 1°C of the adjacent media temperature, with less capital investment than other non-contact-type heat exchangers.
- Plate heat exchangers offer opportunities for energy conservation by regeneration.

As shown in a simple schematic in Figure 4.4, a liquid food is heated to pasteurization or other desired temperature in the heating section; the heated fluid then surrenders part of its heat to the incoming raw fluid in the regeneration section. The cold stream is heated to a temperature where it requires little additional energy to bring it up to the desired



W

Figure 4.4 A five-stage plate pasteurizer for processing milk. (Reprinted with permission of Alfa-Laval AB, Tumba, Sweden, and Alfa-Laval, Inc., Fort Lee, New Jersey)



W

Figure 4.5 A two-way regeneration system used in processing grape juice. (Courtesy of APV Equipment, Inc.)

temperature. For regeneration, additional plates are required; however, the additional capital cost may be recovered quickly by lowered operating costs.

An actual two-way regeneration process is shown in Figure 4.5 for pasteurizing grape juice. After the "starter" juice has been heated to 88°C (at location A), it is passed through a holding loop and into the regenerative section (entering at location B). In this section, the juice releases its heat to incoming raw juice entering (at location C) into the exchanger at 38°C. The temperature of raw juice increases to 73°C (at location D), and the "starter" juice temperature decreases to 53°C (at location E). In this example, the regeneration is [(73−38)/(88−38)] × 100 or 70%, since the incoming raw juice was heated to 70% of its eventual pasteurization temperature without the use of an external heating medium. The juice heated to 73°C passes through the heating section, where its temperature is raised to 88°C by using 93°C hot water as the heating medium. The heated juice is then pumped to the regeneration section, where it preheats the incoming raw juice, and the cycle continues. The cooling of hot pasteurized juice is accomplished by using city water, chilled water, or recirculated glycol. It should be noted that, in this example, less heat needs to be removed from the pasteurized juice, thus decreasing the cooling load by the regeneration process.

4.1.2 Tubular Heat Exchanger

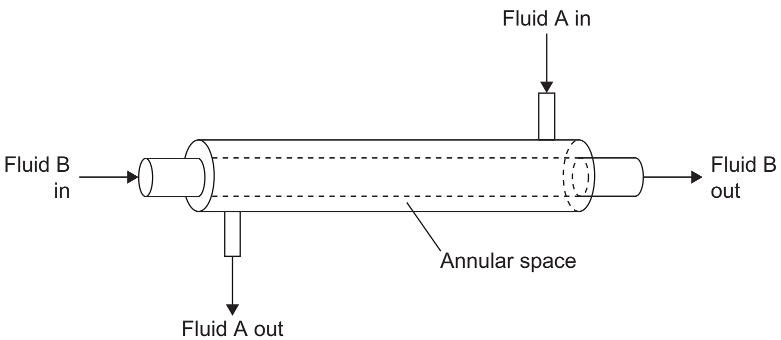
The simplest noncontact-type heat exchanger is a double-pipe heat exchanger, consisting of a pipe located concentrically inside another pipe. The two fluid streams flow in the annular space and in the inner pipe, respectively.

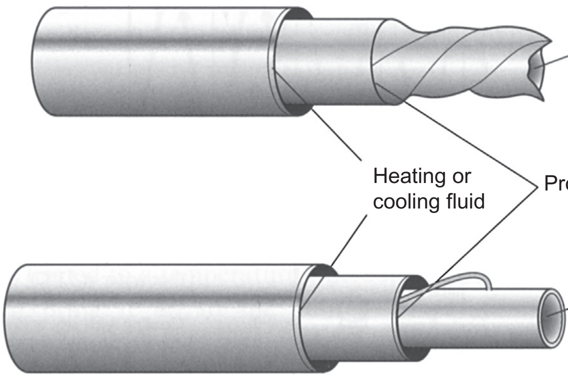
The streams may flow in the same direction (parallel flow) or in the opposite direction (counterflow). Figure 4.6 is a schematic diagram of a counterflow double-pipe heat exchanger.

A slight variation of a double-pipe heat exchanger is a triple-tube heat exchanger, shown in Figure 4.7. In this type of heat exchanger, product flows in the inner annular space, whereas the heating/cooling medium flows in the inner tube and outer annular space. The

W

■ **Figure 4.6** Schematic illustration of a tubular heat exchanger.





W

Figure 4.7 Schematic illustration of a triple-tube heat exchanger. (Courtesy of Paul Mueller Co.)

innermost tube may contain specially designed obstructions to create turbulence and better heat transfer. Some specific industrial applications of triple-tube heat exchangers include heating single-strength orange juice from 4 to 93°C and then cooling to 4°C; cooling cottage cheese wash water from 46 to 18°C with chilled water; and cooling ice cream mix from 12 to 0.5°C with ammonia.

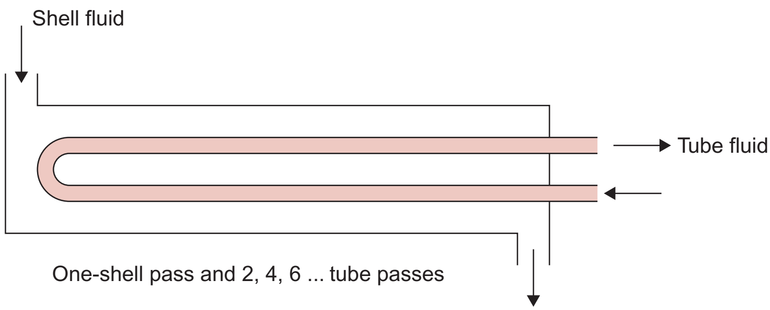
Another common type of heat exchanger used in the food industry is a shell-and-tube heat exchanger for such applications as heating liquid foods in evaporation systems. As shown in Figure 4.8, one of the fluid streams flows inside the tube while the other fluid stream is pumped over the tubes through the shell. By maintaining the fluid stream in the shell side to flow over the tubes, rather than parallel to the tubes, we can achieve higher rates of heat transfer. Baffles located in the shell side allow the cross-flow pattern. One or more tube passes can be accomplished, depending on the design. The shell-and-tube heat exchangers shown in Figure 4.8 are one shell pass with two tube passes, and two shell passes with four tube passes.

4.1.3 Scraped-Surface Heat Exchanger

In conventional types of tubular heat exchangers, heat transfer to a fluid stream is affected by hydraulic drag and heat resistance due to film buildup or fouling on the tube wall. This heat resistance can be minimized if the inside surface of the tube wall is scraped continuously by some mechanical means. The scraping action allows rapid heat transfer to a relatively small product volume. A scraped-surface heat exchanger, used in food processing, is shown schematically in Figure 4.9.

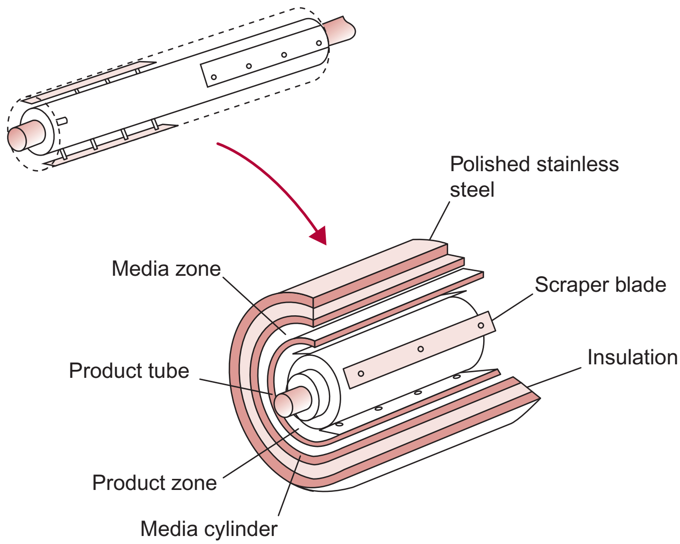
W

■ **Figure 4.8** A shell-and-tube heat exchanger.



W

■ **Figure 4.9** A scraped-surface heat exchanger with a cutaway section illustrating various components. (Courtesy of Cherry-Burrell Corporation)



The food contact areas of a scraped-surface cylinder are fabricated from stainless steel (type 316), pure nickel, hard chromium-plated nickel, or other corrosion-resistant material. The inside rotor contains blades that are covered with plastic laminate or molded plastic (Fig. 4.9). The rotor speed varies between 150 and 500 rpm. Although higher rotation speed allows better heat transfer, it may affect the quality of the processed product by possible maceration. Thus, we must carefully select the rotor speed and the annular space between the rotor and the cylinder for the product being processed.

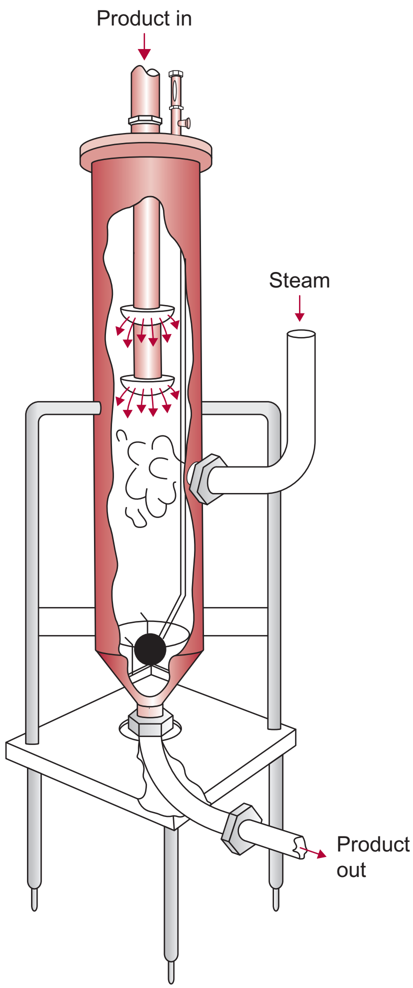
As seen in Figure 4.9, the cylinder containing the product and the rotor is enclosed in an outside jacket. The heating/cooling medium is supplied to this outside jacket. Commonly used media include steam, hot water, brine, or a refrigerant. Typical temperatures used for processing products in scraped-surface heat exchangers range from −35 to 190°C.

The constant blending action accomplished in the scraped-surface heat exchanger is often desirable to enhance the uniformity of product flavor, color, aroma, and textural characteristics. In the food processing industry, the applications of scraped-surface heat exchangers include heating, pasteurizing, sterilizing, whipping, gelling, emulsifying, plasticizing, and crystallizing. Liquids with a wide range of viscosities that can be pumped are processed in these heat exchangers; examples include fruit juices, soups, citrus concentrate, peanut butter, baked beans, tomato paste, and pie fillings.

4.1.4 Steam-Infusion Heat Exchanger

A steam-infusion heat exchanger provides a direct contact between steam and the product. As shown in Figure 4.10, product in liquid state is pumped to the top of the heat exchanger and then allowed to flow in thin sheets in the heating chamber. The viscosity of the liquid determines the size of the spreaders. Products containing particulates, such as diced vegetables, meat chunks, and rice, can be handled by specially designed spreaders. High rates of heat transfer are achieved when steam contacts tiny droplets of the food. The temperature of the product rises very rapidly due to steam condensation. The heated products with condensed steam are released from the chamber at the bottom. A specific amount of liquid is retained in the bottom of the chamber to achieve desired cooking.

The temperature difference of the product between the inlet and the outlet to the heating chamber may be as low as 5.5°C, such as for



W

Figure 4.10 A steam-infusion heat exchanger. (Courtesy of CREPACO, Inc.)

deodorizing milk (76.7 to 82.2°C), or as high as 96.7°C, such as for sterilizing puddings for aseptic packaging (48.9 to 145.6°C).

The water added to the product due to steam condensation is sometimes desirable, particularly if the overall process requires addition of water. Otherwise, the added water of condensation can be "flashed off" by pumping the heated liquid into a vacuum cooling system. The amount of water added due to condensation can be computed by measuring the temperature of the product fed to the heat exchanger and the temperature of the product discharged from the vacuum cooler.

This type of heat exchanger has applications in cooking and/or sterilizing a wide variety of products, such as concentrated soups, chocolate, processed cheese, ice cream mixes, puddings, fruit pie fillings, and milk.

4.1.5 Epilogue

In the preceding subsections, we discussed several types of commonly used heat exchangers. It should be evident that a basic understanding of the mechanisms of heat transfer, both in the food and the materials used in construction of the food processing equipment, is necessary before we can design or evaluate any heat exchange equipment. A wide variety of food products is processed using heat exchangers. These products present unique and often complex problems related to heat transfer. In the following sections, we will develop quantitative descriptions emphasizing the following:

1. *Thermal properties*. Properties such as specific heat, thermal conductivity, and thermal diffusivity of food and equipment materials (such as metals) play an important role in determining the rate of heat transfer.

2. *Mode of heat transfer*. A mathematical description of the actual mode of heat transfer, such as conduction, convection, and/or radiation is necessary to determine quantities, such as total amount of heat transferred from heating or cooling medium to the product.

3. *Steady-state and unsteady-state heat transfer*. Calculation procedures are needed to examine both the unsteady-state and steady-state phases of heat transfer.

We will develop an analytical approach for cases involving simple heat transfer. For more complex treatment of heat transfer, such as for non-Newtonian liquids, the textbook by Heldman and Singh (1981) is recommended.

4.2 THERMAL PROPERTIES OF FOODS

4.2.1 Specific Heat

Specific heat is the quantity of heat that is gained or lost by a unit mass of product to accomplish a unit change in temperature, without a change in state:

$$c_p = \frac{Q}{m(\Delta T)} \tag{4.1}$$

where Q is heat gained or lost (kJ), m is mass (kg), ΔT is temperature change in the material (°C), and c_p is specific heat (kJ/[kg °C]).

Specific heat is an essential part of the thermal analysis of food processing or of the equipment used in heating or cooling of foods. With food materials, this property is a function of the various components that constitute a food, its moisture content, temperature, and pressure. The specific heat of a food increases as the product moisture content increases. For a gas, the specific heat at constant pressure, c_p, is greater than its specific heat at constant volume, c_v. In most food processing applications, we use specific heat at constant pressure c_p, since pressure is generally kept constant except in high-pressure processing.

For processes where a change of state takes place, such as freezing or thawing, an apparent specific heat is used. Apparent specific heat incorporates the heat involved in the change of state in addition to the sensible heat.

In designing food processes and processing equipment, we need numerical values for the specific heat of the food and materials to be used. There are two ways to obtain such values. Published data are available that provide values of specific heat for some food and nonfood materials, such as given in Tables A.2.1, A.3.1, and A.3.2 (in the Appendix). Comprehensive databases are also available to obtain published values (Singh, 1994). Another way to obtain a specific heat value is to use a predictive equation. The predictive equations are empirical expressions, obtained by fitting experimental data into mathematical models. Typically these mathematical models are based on one or more constituents of the food. Since water is a major component of many foods, a number of models are expressed as a function of water content.

One of the earliest models to calculate specific heat was proposed by Siebel (1892) as,

$$c_p = 0.837 + 3.349\,X_w \tag{4.2}$$

where X_w is the water content expressed as a fraction. This model does not show the effect of temperature or other components of a food product. The influence of product components was expressed in an empirical equation proposed by Charm (1978) as

$$c_p = 2.093\,X_f + 1.256\,X_s + 4.187\,X_w \tag{4.3}$$

where X is the mass fraction; and subscripts f is fat, s is nonfat solids, and w is water. Note that in Equation (4.3), the coefficients of each term on the right-hand side are specific heat values of the respective food constituents. For example, 4.187 is the specific heat of water at 70°C, and 2.093 is the specific heat of liquid fat.

Heldman and Singh (1981) proposed the following expression based on the components of a food product:

$$c_p = 1.424\,X_h + 1.549\,X_p + 1.675\,X_f + 0.837\,X_a + 4.187\,X_w \tag{4.4}$$

where X is the mass fraction; the subscripts on the right-hand side are h, carbohydrate; p, protein; f, fat; a, ash; and w, moisture.

Note that these equations do not include a dependence on temperature. However, for processes where temperature changes, we must use predictive models of specific heat that include temperature dependence. Choi and Okos (1986) presented a comprehensive model to predict specific heat based on composition and temperature. Their model is as follows:

$$c_p = \sum_{i=1}^{n} c_{pi} X_i \tag{4.5}$$

where X_i is the fraction of the ith component, n is the total number of components in a food, and c_{pi} is the specific heat of the ith component. Table A.2.9 gives the specific heat of pure food components as a function of temperature. The coefficients in this table may be programmed in a spreadsheet for predicting specific heat at any desired temperature, as illustrated in Example 4.1.

The units for specific heat are

$$c_p = \frac{\text{kJ}}{\text{kg K}}$$

Note that these units are equivalent to kJ/(kg °C), since 1° temperature *change* is the same in Celsius or Kelvin scale.

Food composition values may be obtained from *Agriculture Handbook No. 8* (Watt and Merrill, 1975). Values for selected foods are given in Table A.2.8.

Example 4.1

Predict the specific heat for a model food with the following composition: carbohydrate 40%, protein 20%, fat 10%, ash 5%, moisture 25%.

Given

$X_h = 0.4 \quad X_p = 0.2 \quad X_f = 0.1 \quad X_a = 0.05 \quad X_m = 0.25$

Approach

Since the product composition is given, Equation (4.4) will be used to predict specific heat. Furthermore, we will program a spreadsheet with Equation (4.5) to determine a value for specific heat.

	A	B	C	D	E	F	G	H	I
1	Temperature (°C)	20							
2	Water	0.25							
3	Protein	0.2							
4	Fat	0.1							
5	Carbohydrate	0.4							
6	Fiber	0							
7	Ash	0.05							
8									
9		Coefficients							
10	Water	4.1766		=4.1762−0.000090864*$B$1+0.0000054731*$B$1^2					
11	Protein	2.0319		=2.0082+0.0012089*$B$1−0.0000013129*$B$1^2					
12	Fat	2.0117		=1.9842+0.0014733*$B$1−0.0000048008*$B$1^2					
13	Carbohydrate	1.5857		=1.5488+0.0019625*$B$1−0.0000059399*$B$1^2					
14	Fiber	1.8807		=1.8459+0.0018306*$B$1−0.0000046509*$B$1^2					
15	Ash	1.1289		=1.0926+0.0018896*$B$1−0.0000036817*$B$1^2					
16									
17		Eq(4.5)							
18	Water	1.044		=B2*B10					
19	Protein	0.406		=B3*B11					
20	Fat	0.201		=B4*B12					
21	Carbohydrate	0.634		=B5*B13					
22	Fiber	0.000		=B6*B14					
23	Ash	0.056		=B7*B15					
24	Result	2.342		=SUM(B18:B23)					

■ **Figure E4.1** Spreadsheet for data given in Example 4.1.

Solution

1. *Using Equation (4.4)*

$$c_p = (1.424 \times 0.4) + (1.549 \times 0.2) + (1.675 \times 0.1) + (0.837 \times 0.05) + (4.187 \times 0.25) = 2.14\ \text{kJ}/(\text{kg}\ ^\circ\text{C})$$

2. *We can program a spreadsheet using Equation (4.5) with coefficients given in Table A.2.9 as shown in Figure E4.1.*
3. *Specific heat predicted using Equation (4.4) is 2.14 kJ/(kg °C) whereas using Equation (4.5) is slightly different as 2.34 kJ/(kg °C). Equation (4.5) is preferred since it incorporates information about the temperature.*

4.2.2 Thermal Conductivity

The thermal conductivity of a food is an important property used in calculations involving rate of heat transfer. In quantitative terms, this property gives the amount of heat that will be conducted per unit time through a unit thickness of the material if a unit temperature gradient exists across that thickness.

In SI units, thermal conductivity is

$$k \equiv \frac{\text{J}}{\text{s}\ m\ ^\circ C} \equiv \frac{\text{W}}{m\ ^\circ C} \tag{4.6}$$

Note that W/(m °C) is same as W/(m K).

There is wide variability in the magnitude of thermal conductivity values for commonly encountered materials. For example:

- Metals: 50–400 W/(m °C)
- Alloys: 10–120 W/(m °C)
- Water: 0.597 W/(m °C) (at 20°C)
- Air: 0.0251 W/(m °C) (at 20°C)
- Insulating materials: 0.035–0.173 W/(m °C)

Most high-moisture foods have thermal conductivity values closer to that of water. On the other hand, the thermal conductivity of dried, porous foods is influenced by the presence of air with its low value. Tables A.2.2, A.3.1, and A.3.2 show thermal conductivity values obtained numerically for a number of food and nonfood materials. In addition to the tabulated values, empirical predictive equations are useful in process calculations where temperature may be changing.

For fruits and vegetables with a water content greater than 60%, the following equation has been proposed (Sweat, 1974):

$$k = 0.148 + 0.493\,X_{\mathrm{w}} \tag{4.7}$$

where k is thermal conductivity (W/[m °C]), and X_{w} is water content expressed as a fraction. For meats and fish, temperature 0–60°C, water content 60–80%, wet basis, Sweat (1975) proposed the following equation:

$$k = 0.08 + 0.52\,X_{\mathrm{w}} \tag{4.8}$$

Another empirical equation developed by Sweat (1986) is to fit a set of 430 data points for solid and liquid foods, as follows:

$$k = 0.25\,X_{\mathrm{h}} + 0.155\,X_{\mathrm{p}} + 0.16\,X_{\mathrm{f}} + 0.135\,X_{\mathrm{a}} + 0.58\,X_{\mathrm{w}} \tag{4.9}$$

where X is the mass fraction, and subscript h is carbohydrate, p is protein, f is fat, a is ash, and w is water.

The coefficients in Equation (4.9) are thermal conductivity values of the pure component. Note that the thermal conductivity of pure water at 25°C is 0.606 W/(m °C). The coefficient of 0.58 in Equation (4.9) indicates that there is either a bias in the data set used for regression, or the effective thermal conductivity of water in a food is different from that of pure water.

Equations (4.7) to (4.9) are simple expressions to calculate the thermal conductivity of foods, however they do not include the influence of temperature. Choi and Okos (1986) gave the following expression that includes the influence of product composition and temperature:

$$k = \sum_{i=1}^{n} k_i Y_i \tag{4.10}$$

where a food material has n components, k_i is the thermal conductivity of the ith component, Y_i is the volume fraction of the ith component, obtained as follows:

$$Y_i = \frac{X_i/\rho_i}{\sum_{i=1}^{n}(X_i/\rho_i)} \tag{4.11}$$

where X_i is the weight fraction and ρ_i is the density (kg/m^3) of the ith component.

The coefficients for k_i for pure components are listed in Table A.2.9. They may be programmed into a spreadsheet, as illustrated later in Example 4.2.

For the additive models, Equations (4.10) and (4.11), the food composition values may be obtained from Table A.2.8. These equations predict thermal conductivity of foods within 15% of experimental values.

In the case of anisotropic foods, the properties of the material are direction dependent. For example, the presence of fibers in beef results in different values of thermal conductivity when measured parallel to the fibers (0.476 W/[m °C]) versus perpendicular to them (0.431 W/[m °C]). Mathematical models to predict the thermal conductivity of anisotropic foods are discussed in Heldman and Singh (1981).

4.2.3 Thermal Diffusivity

Thermal diffusivity, a ratio involving thermal conductivity, density, and specific heat, is given as,

$$\alpha = \frac{k}{\rho c_{\mathrm{p}}} \tag{4.12}$$

The units of thermal diffusivity are

$$\alpha \equiv \frac{m^2}{s}$$

Thermal diffusivity may be calculated by substituting values of thermal conductivity, density, and specific heat in Equation (4.12). Table A.2.3 gives some experimentally determined values of thermal diffusivity. Choi and Okos (1986) provided the following predictive equation, obtained by substituting the values of k, ρ, and c_{p} in Equation (4.12):

$$\alpha = \sum_{i=1}^{n} \alpha_i X_i \tag{4.13}$$

where n is the number of components, α_i is the thermal diffusivity of the ith component, and X_i is the mass fraction of each component. The values of α_i are obtained from Table A.2.9.

Example 4.2

Estimate the thermal conductivity of hamburger beef that contains 68.3% water.

Given

$X_m = 0.683$

Approach

We will use Equation (4.8), which is recommended for meats. We will also program a spreadsheet using Equations (4.10) and (4.11) at 20°C to calculate thermal conductivity.

Solution

1. *Using Equation (4.8)*

$$k = 0.08 + (0.52 \times 0.683)$$
$$= 0.435 \ W/(m \ ^\circ C)$$

2. *Next we will program a spreadsheet as shown in Figure E4.2 using the composition of hamburger beef from Table A.2.8 and coefficients of Equations (4.10) and (4.11) given in Table A.2.9. We will use a temperature of 20°C.*

	A	B	C	D	E	F	G	H	I
1									
2	Given								
3	Temperature (°C)	20							
4	Water	0.683							
5	Protein	0.207							
6	Fat	0.1							
7	Carbohydrate	0							
8	Fiber	0	=997.18+0.0031439*$B$3−0.0037574*$B$3^2						
9	Ash	0.01							
10				=B4/B12					
11		density coeff	Xi/ri	Yi					
12	Water	995.739918	0.000686	0.717526					
13	Protein	1319.532	0.000157	0.164102	=C12/$C$18				
14	Fat	917.2386	0.000109	0.114046					
15	Carbohydrate	1592.8908	0.000000	0					
16	Fiber	1304.1822	0.000000	0					
17	Ash	2418.1874	0.000004	0.004326					
18		sum	0.000956						
19				=0.57109+0.0017625*$B$3−0.0000067036*$B$3^2					
20		k Coeff							
21	Water	0.6037	0.4331						
22	Protein	0.2016	0.0331		=B21*D12				
23	Fat	0.1254	0.0143						
24	Carbohydrate	0.2274	0.0000						
25	Fiber	0.2070	0.0000						
26	Ash	0.3565	0.0015						
27									
28	Result		0.4821						

Figure E4.2 Spreadsheet for data given in Example 4.2.

3. *The thermal conductivity predicted by Equation (4.8) is 0.435 W/(m °C), whereas using Equation (4.10) it is 0.4821 W/(m °C). Although Equation (4.8) is easier to use, it does not include the influence of temperature.*

4.3 MODES OF HEAT TRANSFER

In Chapter 1, we reviewed various forms of energy, such as thermal, potential, mechanical, kinetic, and electrical. Our focus in this chapter will be on thermal energy, commonly referred to as heat energy or heat content. As noted in Section 1.19, heat energy is simply the sensible and latent forms of internal energy. Recall that the heat content of an object such as a tomato is determined by its mass, specific heat, and temperature. The equation for calculating heat content is

$$Q = mc_p \Delta T \tag{4.14}$$

where m is mass (kg), c_p is specific heat at constant pressure (kJ/[kg K]), and ΔT is the temperature difference between the object and a reference temperature (°C). Heat content is always expressed relative to some other temperature (called a datum or reference temperature).

Although determining heat content is an important calculation, the knowledge of how heat may transfer from one object to another or within an object is of even greater practical value. For example, to thermally sterilize tomato juice, we raise its heat content by transferring heat from some heating medium such as steam into the juice. In order to design the sterilization equipment, we need to know how much heat is necessary to raise the temperature of tomato juice from the initial to the final sterilization temperature using Equation (4.14). Furthermore, we need to know the rate at which heat will transfer from steam into the juice first passing through the walls of the sterilizer. Therefore, our concerns in heating calculations are twofold: the quantity of heat transferred, Q, expressed in the units of joule (J); and the rate of heat transfer, q, expressed as joule/s (J/s) or watt (W).

We will first review some highlights of the three common modes of heat transfer—conduction, convection, and radiation—and then examine selected topics of rates of heat transfer important in the design and analysis of food processes.

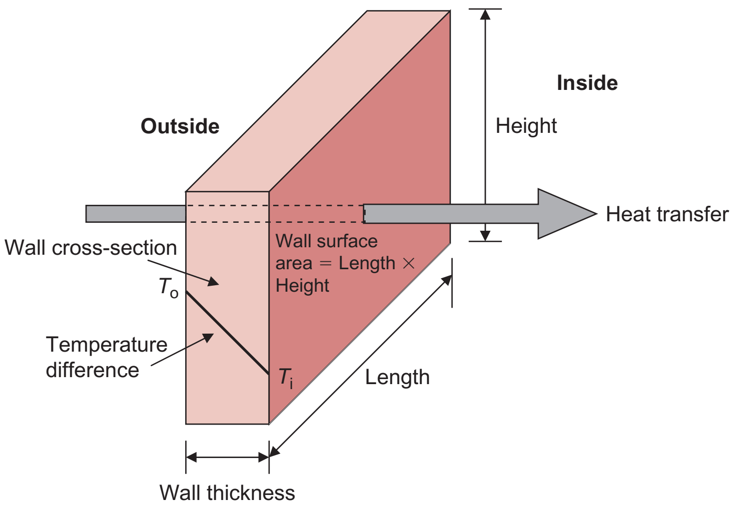
4.3.1 Conductive Heat Transfer

Conduction is the mode of heat transfer in which the transfer of energy takes place at a molecular level. There are two commonly accepted theories that describe conductive heat transfer. According to one theory, as molecules of a solid material attain additional thermal energy, they become more energetic and vibrate with increased amplitude of vibration while confined in their lattice. These vibrations are transmitted from one molecule to another without actual translatory motion of the molecules. Heat is thus conducted from regions of higher temperature to those at lower temperature. The second theory states that conduction occurs at a molecular level due to the drift of free electrons. These free electrons are prevalent in metals, and they carry thermal and electrical energy. For this reason, good conductors of electricity such as silver and copper are also good conductors of thermal energy.

Note that in conductive mode, there is no physical movement of the object undergoing heat transfer. Conduction is the common mode of heat transfer in heating/cooling of opaque solid materials.

From everyday experience, we know that on a hot day, heat transfer from the outside to the inside through the wall of a room (Fig. 4.11) depends on the surface area of the wall (a wall with larger surface area will conduct more heat), the thermal properties of construction materials (steel will conduct more heat than brick), wall thickness (more heat transfer through a thin wall than thick), and temperature difference (more heat transfer will occur when the outside temperature is much hotter than the inside room temperature). In other words, the rate of heat transfer through the wall may be expressed as

$$q \propto \frac{(\text{wall surface area})(\text{temperature difference})}{(\text{wall thickness})} \tag{4.15}$$



■ **Figure 4.11** Conductive heat flow in a wall.

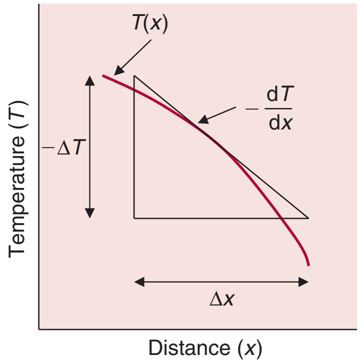


Figure 4.12 Sign convention for conductive heat flow.

or

$$q_x \propto \frac{A\,dT}{dx} \tag{4.16}$$

or, by inserting a constant of proportionality,

$$q_x = -kA\frac{dT}{dx} \tag{4.17}$$

where q_x is the rate of heat flow in the direction of heat transfer by conduction (W); k is thermal conductivity (W/[m °C]); A is area (normal to the direction of heat transfer) through which heat flows (m^2); T is temperature (°C); and x is length (m), a variable.

Equation (4.17) is also called Fourier's law for heat conduction, after Joseph Fourier, a French mathematical physicist. According to the second law of thermodynamics, heat will always conduct from higher temperature to lower temperature. As shown in Figure 4.12, the gradient dT/dx is negative, because temperature decreases with increasing values of x. Therefore, in Equation (4.17), a negative sign is used to obtain a positive value for heat flow in the direction of decreasing temperature.

Example 4.3

One face of a stainless-steel plate 1 cm thick is maintained at 110°C, and the other face is at 90°C (Fig. E4.3). Assuming steady-state conditions, calculate the rate of heat transfer per unit area through the plate. The thermal conductivity of stainless steel is 17 W/(m °C).

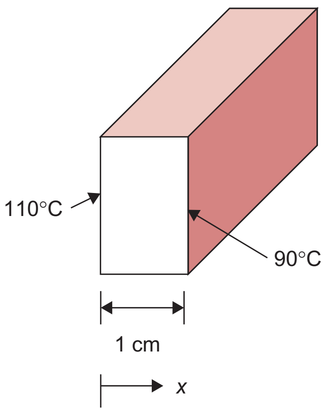


Figure E4.3 Heat flow in a plate.

Given

Thickness of plate = 1 cm = 0.01 m
Temperature of one face = 110°C
Temperature of other face = 90°C
Thermal conductivity of stainless steel = 17 W/(m °C)

Approach

For steady-state heat transfer in rectangular coordinates we will use Equation (4.17) to compute rate of heat transfer.

Solution

1. *From Equation (4.17)*

$$q = -\frac{17\,[W/(m\,°C)] \times 1\,[m^2] \times (110 - 90)\,[°C]}{(0 - 0.01)\,[m]}$$

$$= 34,000\ W$$

2. *Rate of heat transfer per unit area is calculated to be 34,000 W. A positive sign is obtained for the heat transfer, indicating that heat always flows "downhill" from 110°C to 90°C.*

4.3.2 Convective Heat Transfer

When a fluid (liquid or gas) comes into contact with a solid body such as the surface of a wall, heat exchange will occur between the solid and the fluid whenever there is a temperature difference between the two. During heating and cooling of gases and liquids the fluid streams exchange heat with solid surfaces by convection.

The magnitude of the fluid motion plays an important role in convective heat transfer. For example, if air is flowing at a high velocity past a hot baked potato, the latter will cool down much faster than if the air velocity was much lower. The complex behavior of fluid flow next to a solid surface, as seen in velocity profiles for laminar and turbulent flow conditions in Chapter 2, make the determination of convective heat transfer a complicated topic.

Depending on whether the flow of the fluid is artificially induced or natural, there are two types of convective heat transfer: **forced** convection and **free** (also called **natural**) convection. Forced convection involves the use of some mechanical means, such as a pump or a fan, to induce movement of the fluid. In contrast, free convection occurs due to density differences caused by temperature gradients within the system. Both of these mechanisms may result in either laminar or turbulent flow of the fluid, although turbulence occurs more often in forced convection heat transfer.

Consider heat transfer from a heated flat plate, PQRS, exposed to a flowing fluid, as shown in Figure 4.13. The surface temperature of the plate is T_s, and the temperature of the fluid far away from the plate surface is T_∞. Because of the viscous properties of the fluid, a velocity profile is set up within the flowing fluid, with the fluid velocity decreasing to zero at the solid surface. Overall, we see that the rate of heat transfer from the solid surface to the flowing fluid is proportional to the surface area of solid, A, in contact with the fluid, and the difference between the temperatures T_s and T_∞. Or,

$$q \propto A(T_s - T_\infty) \qquad (4.18)$$

or,

$$q = hA(T_s - T_\infty) \qquad (4.19)$$

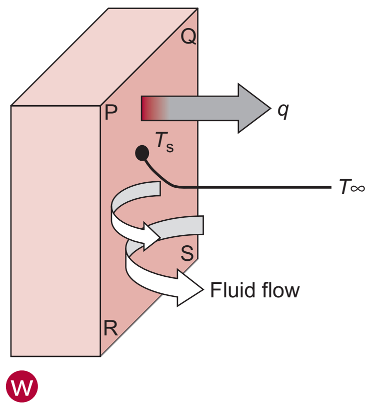


Figure 4.13 Convective heat flow from the surface of a flat plate.

Table 4.1 Some Approximate Values of Convective Heat-Transfer Coefficient

Fluid	Convective heat-transfer coefficient (W/[m^2 K])
Air	
Free convection	5–25
Forced convection	10–200
Water	
Free convection	20–100
Forced convection	50–10,000
Boiling water	3000–100,000
Condensing water vapor	5000–100,000

The area is A (m^2), and h is the convective heat-transfer coefficient (sometimes called surface heat-transfer coefficient), expressed as W/(m^2 °C). This equation is also called Newton's law of cooling.

Note that the convective heat transfer coefficient, h, is not a property of the solid material. This coefficient, however, depends on a number of properties of fluid (density, specific heat, viscosity, thermal conductivity), the velocity of fluid, geometry, and roughness of the surface of the solid object in contact with the fluid. Table 4.1 gives some approximate values of h. A high value of h reflects a high rate of heat transfer. Forced convection offers a higher value of h than free convection. For example, you feel cooler sitting in a room with a fan blowing air than in a room with stagnant air.

Example 4.4

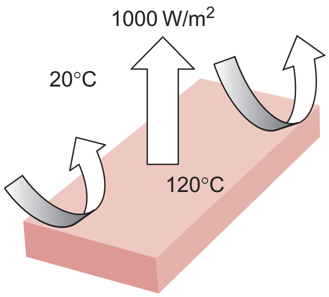


Figure E4.4 Convective heat transfer from a plate.

The rate of heat transfer per unit area from a metal plate is 1000 W/m^2. The surface temperature of the plate is 120°C, and ambient temperature is 20°C (Fig. E4.4). Estimate the convective heat transfer coefficient.

Given

Plate surface temperature = 120°C

Ambient temperature = 20°C

Rate of heat transfer per unit area = 1000 W/m^2

Approach

Since the rate of heat transfer per unit area is known, we will estimate the convective heat transfer coefficient directly from Newton's law of cooling, Equation (4.19).

Solution

1. *From Equation (4.19),*

$$h = \frac{1000\,[W/m^2]}{(120 - 20)\,[°C]}$$
$$= 10\ W/(m^2\ °C)$$

2. *The convective heat transfer coefficient is found to be 10 W/(m^2 °C).*

4.3.3 Radiation Heat Transfer

Radiation heat transfer occurs between two surfaces by the emission and later absorption of electromagnetic waves (or photons). In contrast to conduction and convection, radiation requires no physical medium for its propagation—it can even occur in a perfect vacuum, moving at the speed of light, as we experience everyday solar radiation. Liquids are strong absorbers of radiation. Gases are transparent to radiation, except that some gases absorb radiation of a particular wavelength (for example, ozone absorbs ultraviolet radiation). Solids are opaque to thermal radiation. Therefore, in problems involving thermal radiation with solid materials, such as with solid foods, our analysis is concerned primarily with the surface of the material. This is in contrast to microwave and radio frequency radiation, where the wave penetration into a solid object is significant.

All objects at a temperature above 0 Absolute emit thermal radiation. Thermal radiation emitted from an object's surface is proportional to the absolute temperature raised to the fourth power and the surface characteristics. More specifically, the rate of heat emission (or radiation) from an object of a surface area A is expressed by the following equation:

$$q = \sigma \varepsilon A T_{\mathrm{A}}^4 \tag{4.20}$$

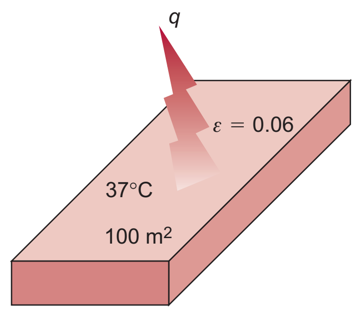
where σ is the Stefan–Boltzmann[1] constant, equal to 5.669×10^{-8} W/(m^2 K^4); T_A is temperature, Absolute; A is the area (m^2); and

[1] Josef Stefan (1835–1893). An Austrian physicist, Stefan began his academic career at the University of Vienna as a lecturer. In 1866, he was appointed director of the Physical Institute. Using empirical approaches, he derived the law describing radiant energy from blackbodies. Five years later, another Austrian, Ludwig Boltzmann, provided the thermodynamic basis of what is now known as the Stefan–Boltzmann law.

ε is emissivity, which describes the extent to which a surface is similar to a blackbody. For a blackbody, the value of emissivity is 1. Table A.3.3 gives values of emissivity for selected surfaces.

Example 4.5

Calculate the rate of heat energy emitted by 100 m^2 of a polished iron surface (emissivity = 0.06) as shown in Figure E4.5. The temperature of the surface is 37°C.



■ **Figure E4.5** Heat transfer from a plate.

Given

Emissivity $\varepsilon = 0.06$
Area $A = 100\ m^2$
Temperature $= 37°C = 310\ K$

Approach

We will use the Stefan–Boltzmann law, Equation (4.20), to calculate the rate of heat transfer due to radiation.

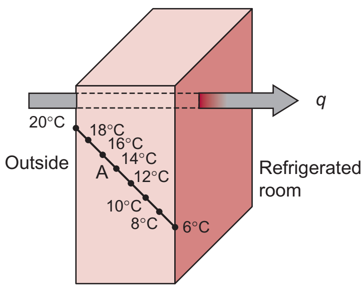
Solution

1. *From Equation (4.20)*

$$q = (5.669 \times 10^{-8}\ W/[m^2\ K^4])(0.06)(100\ m^2)(310\ K)^4$$
$$= 3141\ W$$

2. *The total energy emitted by the polished iron surface is 3141 W.*

4.4 STEADY-STATE HEAT TRANSFER



■ **Figure 4.14** Steady state conductive heat transfer in a wall.

In problems involving heat transfer, we often deal with steady state and unsteady state (or transient) conditions. **Steady-state** conditions imply that time has no influence on the temperature distribution within an object, although temperature may be different at different locations within the object. Under **unsteady-state** conditions, the temperature changes with location and time. For example, consider the wall of a refrigerated warehouse as shown in Figure 4.14. The inside wall temperature is maintained at 6°C using refrigeration, while the outside wall temperature changes throughout the day and night. Assume that for a few hours of the day, the outside wall temperature is constant at 20°C, and during that time duration the rate of heat transfer into the warehouse through the wall will be under steady-state conditions. The temperature at any location inside the wall cross-section (e.g., 14°C at location A) will remain constant, although this temperature is different from other locations along the path of heat transfer within the wall, as shown in Figure 4.14. If, however, the temperature of the outside wall surface changes (say, increases above 20°C), then the heat transfer through the wall will be due to unsteady-state conditions, because now the temperature within the wall will change with time and location. Although true steady-state conditions are uncommon, their mathematical analysis is much simpler. Therefore, if appropriate, we assume steady-state conditions for the analysis of a given problem to obtain useful information for designing equipment and processes. In certain food processes such as in heating cans for food sterilization, we cannot use steady-state conditions, because the duration of interest is when the temperature is changing rapidly with time, and microbes are being killed. For analyzing those types of problems, an analysis involving unsteady-state heat transfer is used, as discussed later in Section 4.5.

Another special case of heat transfer involves change in temperature inside an object with time but not with location, such as might occur during heating or cooling of a small aluminum sphere, which has a high thermal conductivity. This is called a **lumped system**. We will discuss this case in more detail in Section 4.5.2.

In the following section, we will examine several applications of steady-state conduction heat transfer.

4.4.1 Conductive Heat Transfer in a Rectangular Slab

Consider a slab of constant cross-sectional area, as shown in Figure 4.15. The temperature, T_1, on side X is known. We will develop an equation to determine temperature, T_2, on the opposite side Y and at any location inside the slab under steady-state conditions.

This problem is solved by first writing Fourier's law,

$$q_x = -kA\frac{dT}{dx} \tag{4.21}$$

The boundary conditions are

$$\begin{aligned} x &= x_1 \quad T = T_1 \\ x &= x_2 \quad T = T_2 \end{aligned} \tag{4.22}$$

Separating variables in Equation (4.21), we get

$$\frac{q_x}{A}dx = -kdT \tag{4.23}$$

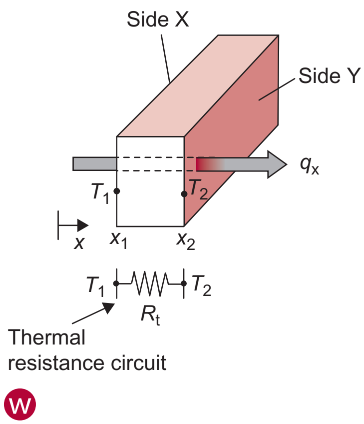


Figure 4.15 Heat transfer in a wall, also shown with a thermal resistance circuit.

Setting up integration and substituting limits, we have

$$\int_{x_1}^{x_2} \frac{q_x}{A} \mathrm{d}x = -\int_{T_1}^{T_2} k \mathrm{d}T \tag{4.24}$$

Since q_x and A are independent of x, and k is assumed to be independent of T, Equation (4.24) can be rearranged to give

$$\frac{q_x}{A} \int_{x_1}^{x_2} \mathrm{d}x = -k \int_{T_1}^{T_2} \mathrm{d}T \tag{4.25}$$

Finally, integrating this equation, we get

$$\frac{q_x}{A}(x_2 - x_1) = -k(T_2 - T_1) \tag{4.26}$$

or

$$q_x = -kA \frac{(T_2 - T_1)}{(x_2 - x_1)} \tag{4.27}$$

Temperature on face Y is T_2; thus, rearranging Equation (4.27),

$$T_2 = T_1 - \frac{q_x}{kA}(x_2 - x_1) \tag{4.28}$$

To determine temperature, T, at any location, x, within the slab, we may replace T_2 and x_2 with unknown T and distance variable x, respectively, in Equation (4.28) and obtain,

$$T = T_1 - \frac{q_x}{kA}(x - x_1) \tag{4.29}$$

4.4.1.1 *Thermal Resistance Concept*

We noted in Chapter 3 that, according to Ohm's Law, electrical current, I, is directly proportional to the voltage difference, E_V, and indirectly proportional to the electrical resistance R_E. Or,

$$I = \frac{E_V}{R_E} \tag{4.30}$$

If we rearrange the terms in Equation (4.27), we obtain

$$q_x = \frac{(T_1 - T_2)}{\left[\dfrac{(x_2 - x_1)}{kA}\right]} \tag{4.31}$$

or,

$$q_x = \frac{T_1 - T_2}{R_t} \tag{4.32}$$

Comparing Equations (4.30) and (4.32), we note an analogy between rate of heat transfer, q_x, and electrical current, I, temperature difference, $(T_1 - T_2)$ and electrical voltage, E_v, and thermal resistance, R_t, and electrical resistance, R_E. From Equations (4.31) and (4.32), thermal resistance may be expressed as

$$R_t = \frac{(x_2 - x_1)}{kA} \tag{4.33}$$

A thermal resistance circuit for a rectangular slab is also shown in Figure 4.15. In solving problems involving conductive heat transfer in a rectangular slab using this concept, we first obtain thermal resistance using Equation (4.33) and then substitute it in Equation (4.32). The rates of heat transfer across the two surfaces of a rectangular slab are thus obtained. This procedure is illustrated in Example 4.6. The advantage of using the thermal resistance concept will become clear when we study conduction in multilayer walls. Moreover, the mathematical computations will be much simpler compared with alter-native procedures used in solving these problems.

Example 4.6

a. Redo Example 4.3 using the thermal resistance concept.

b. Determine the temperature at 0.5 cm from the 110°C temperature face.

Given

See Example 4.3

Location at which temperature is desired = 0.5 cm = 0.005 m

Approach

We will use Equation (4.33) to calculate thermal resistance, and then Equation (4.32) to determine the rate of heat transfer. To determine temperature within the slab, we will calculate the thermal resistance for the thickness of the slab bounded

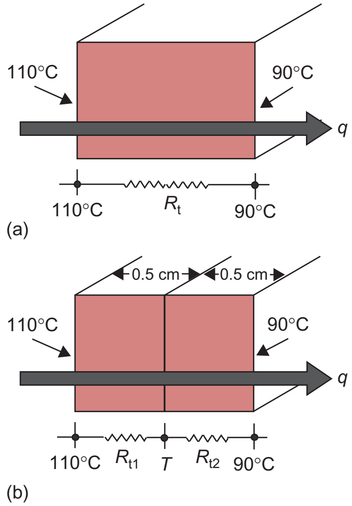


Figure E4.6 Thermal resistance circuits for heat transfer through a wall.

by 110°C and the unknown temperature (Fig. E4.6). Since the steady-state heat transfer remains the same throughout the slab, we will use the previously calculated value of q to determine the unknown temperature using Equation (4.32).

Solution

Part (a)

1. *Using Equation (4.33), the thermal resistance R_t is*

$$R_t = \frac{0.01\,[m]}{17\,[W/(m\,°C)] \times 1\,[m^2]}$$

$$R_t = 5.88 \times 10^{-4}\,°C/W$$

2. *Using Equation (4.32), we obtain rate of heat transfer as*

$$q = \frac{110\,[°C] - 90\,[°C]}{5.88 \times 10^{-4}\,[°C/W]}$$

or

$$q = 34{,}013\ W$$

Part (b)

3. *Using Equation (4.33) calculate resistance R_{t1}*

$$R_{t1} = \frac{0.005\,[m]}{17\,[W/(m\,°C)] \times 1\,[m^2]}$$

$$R_{t1} = 2.94 \times 10^{-4}\,°C/W$$

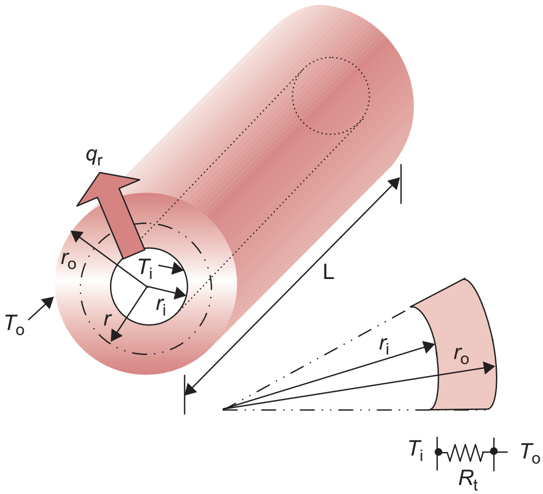
4. *Rearranging terms in Equation (4.32) to determine the unknown temperature T*

$$T = T_1 - (q \times R_{t1})$$
$$T = 110\,[°C] - 34{,}013\,[W] \times 2.94 \times 10^{-4}\,[°C/W]$$
$$T = 100°C$$

5. *The temperature at the midplane is 100°C. This temperature was expected, since the thermal conductivity is constant, and the temperature profile in the steel slab is linear.*

4.4.2 Conductive Heat Transfer through a Tubular Pipe

Consider a long, hollow cylinder of inner radius r_i, outer radius r_o, and length L, as shown in Figure 4.16. Let the inside wall temperature be T_i and the outside wall temperature be T_o. We want to calculate the



W

Figure 4.16 Heat transfer in a radial direction in a pipe, also shown with a thermal resistance circuit.

rate of heat transfer along the radial direction in this pipe. Assume thermal conductivity of the metal remains constant with temperature.

Fourier's law in cylindrical coordinates may be written as

$$q_r = -kA\frac{dT}{dr} \tag{4.34}$$

where q_r is the rate of heat transfer in the radial direction.

Substituting for circumferential area of the pipe,

$$q_r = -k(2\pi rL)\frac{dT}{dr} \tag{4.35}$$

The boundary conditions are

$$\begin{aligned} T &= T_i \quad r = r_i \\ T &= T_o \quad r = r_o \end{aligned} \tag{4.36}$$

Rearranging Equation (4.35), and setting up the integrals,

$$\frac{q_r}{2\pi L}\int_{r_i}^{r_o}\frac{dr}{r} = -k\int_{T_i}^{T_o} dT \tag{4.37}$$

Equation (4.37) gives

$$\frac{q_r}{2\pi L}|\ln r|_{r_i}^{r_o} = -k|T|_{T_i}^{T_o} \tag{4.38}$$

$$q_r = \frac{2\pi Lk(T_i - T_o)}{\ln(r_o/r_i)} \tag{4.39}$$

Again, we can use the electrical resistance analogy to write an expression for thermal resistance in the case of a cylindrical-shaped object. Rearranging the terms in Equation (4.39), we obtain

$$q_r = \frac{(T_i - T_o)}{\left[\dfrac{\ln(r_o/r_i)}{2\pi Lk}\right]} \tag{4.40}$$

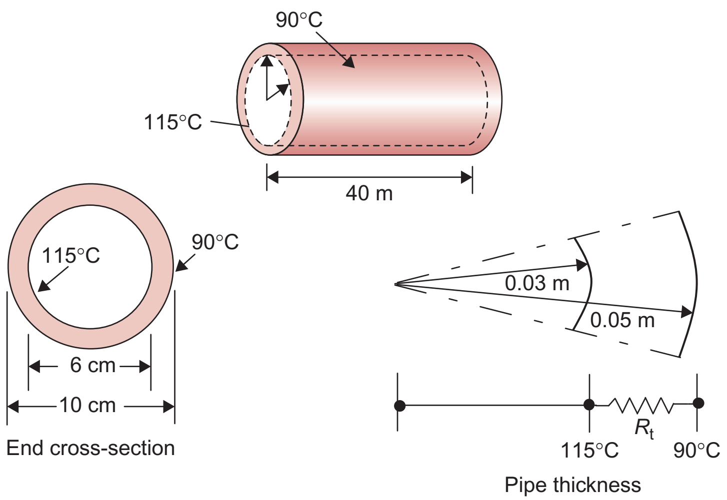
Comparing Equation (4.40) with Equation (4.32), we obtain the thermal resistance in the radial direction for a cylinder as

$$R_t = \frac{\ln(r_o/r_i)}{2\pi Lk} \tag{4.41}$$

Figure 4.16 shows a thermal circuit to obtain R_t. An illustration of the use of this concept is given in Example 4.7.

Example 4.7

A 2-cm-thick steel pipe (thermal conductivity = 43 W/[m °C]) with 6 cm inside diameter is being used to convey steam from a boiler to process equipment for a distance of 40 m. The inside pipe surface temperature is 115°C, and the outside pipe surface temperature is 90°C (Fig. E4.7). Calculate the total heat loss to the surroundings under steady-state conditions.



■ **Figure E4.7** Thermal resistance circuit for heat transfer through a pipe.

Given

Thickness of pipe = 2 cm = 0.02 m

Inside diameter = 6 cm = 0.06 m

Thermal conductivity $k = 43$ W/(m °C)

Length $L = 40$ m

Inside temperature $T_i = 115°C$

Outside temperature $T_o = 90°C$

Approach

We will determine the thermal resistance in the cross-section of the pipe and then use it to calculate the rate of heat transfer, using Equation (4.40).

Solution

1. *Using Equation (4.41)*

$$R_t = \frac{\ln(0.05/0.03)}{2\pi \times 40[\mathrm{m}] \times 43[\mathrm{W/(m\,°C)}]}$$

$$= 4.727 \times 10^{-5}\ °\mathrm{C/W}$$

2. *From Equation (4.40)*

$$q = \frac{115[°\mathrm{C}] - 90[°\mathrm{C}]}{4.727 \times 10^{-5}[°\mathrm{C/W}]}$$

$$= 528{,}903\ \mathrm{W}$$

3. *The total heat loss from the 40 m long pipe is 528,903 W.*

4.4.3 Heat Conduction in Multilayered Systems

4.4.3.1 *Composite Rectangular Wall (in Series)*

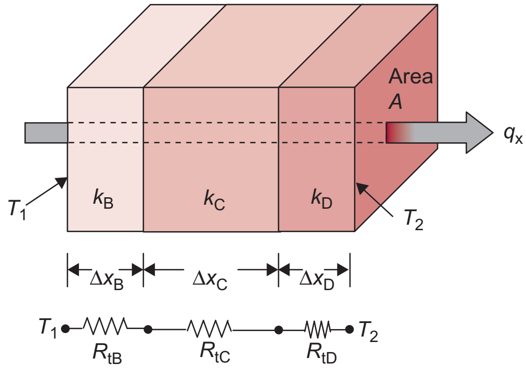
We will now consider heat transfer through a composite wall made of several materials of different thermal conductivities and thicknesses. An example is a wall of a cold storage, constructed of different layers of materials of different insulating properties. All materials are arranged in series in the direction of heat transfer, as shown in Figure 4.17.

From Fourier's law,

$$q = -kA\frac{dT}{dx}$$

This may be rewritten as

$$\Delta T = -\frac{q\Delta x}{kA} \tag{4.42}$$



W

Figure 4.17 Conductive heat transfer in a composite rectangular wall, also shown with a thermal resistance circuit.

Thus, for materials B, C, and D, we have

$$\Delta T_B = -\frac{q\Delta x_B}{k_B A} \quad \Delta T_C = -\frac{q\Delta x_C}{k_C A} \quad \Delta T_D = -\frac{q\Delta x_D}{k_D A} \tag{4.43}$$

From Figure 4.17,

$$\Delta T = T_1 - T_2 = \Delta T_B + \Delta T_C + \Delta T_D \tag{4.44}$$

From Equations (4.42), (4.43), and (4.44),

$$T_1 - T_2 = -\left(\frac{q\Delta x_B}{k_B A} + \frac{q\Delta x_C}{k_C A} + \frac{q\Delta x_D}{k_D A}\right) \tag{4.45}$$

or, rearranging the terms,

$$T_1 - T_2 = -\frac{q}{A}\left(\frac{\Delta x_B}{k_B} + \frac{\Delta x_C}{k_C} + \frac{\Delta x_D}{k_D}\right) \tag{4.46}$$

We can rewrite Equation (4.46) for thermal resistance as

$$q = \frac{T_2 - T_1}{\left(\frac{\Delta x_B}{k_B A} + \frac{\Delta x_C}{k_C A} + \frac{\Delta x_D}{k_D A}\right)} \tag{4.47}$$

or, using thermal resistance values for each layer, we can write Equation (4.47) as,

$$q = \frac{T_2 - T_1}{R_{tB} + R_{tC} + R_{tD}} \tag{4.48}$$

where

$$R_{tB} = \frac{\Delta x_B}{k_B A} \quad R_{tC} = \frac{\Delta x_C}{k_C A} \quad R_{tD} = \frac{\Delta x_D}{k_D A}$$

The thermal circuit for a multilayer rectangular system is shown in Figure 4.17. Example 4.8 illustrates the calculation of heat transfer through a multilayer wall.

Example 4.8

A cold storage wall (3 m × 6 m) is constructed of 15-cm-thick concrete (thermal conductivity = 1.37 W/[m °C]). Insulation must be provided to maintain a heat transfer rate through the wall at or below 500 W (Fig. E4.8). If the thermal conductivity of the insulation is 0.04 W/(m °C), compute the required thickness of the insulation. The outside surface temperature of the wall is 38°C, and the inside wall temperature is 5°C.

Given

Wall dimensions = 3 m × 6 m
Thickness of concrete wall = 15 cm = 0.15 m
$k_{concrete}$ *= 1.37 W/(m °C)*
Maximum heat gain permitted, q = 500 W
$k_{insulation}$ *= 0.04 W/(m °C)*
Outer wall temperature = 38°C
Inside wall (concrete/insulation) temperature = 5°C

Approach

In this problem we know the two surface temperatures and the rate of heat transfer through the composite wall, therefore, using this information we will first calculate the thermal resistance in the concrete layer. Then we will calculate the thermal resistance in the insulation layer, which will yield the thickness value.

Solution

1. *Using Equation (4.48)*

$$q = \frac{(38-5)[°C]}{R_{t1} + R_{t2}}$$

2. *Thermal resistance in the concrete layer,* R_{t2} *is*

$$R_{t2} = \frac{0.15\,[m]}{1.37\,[W/(m\,°C)] \times 18\,[m^2]}$$

$$R_{t2} = 0.0061°C/W$$

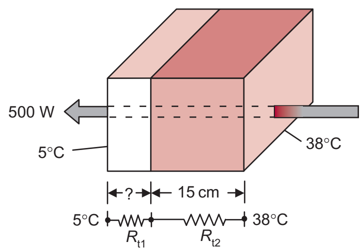


Figure E4.8 Heat transfer through a two-layered wall.

3. *From step 1,*

$$\frac{(38-5)[°C]}{R_{t1}+0.0061[°C/W]} = 500$$

or,

$$R_{t1} = \frac{(38-5)[°C]}{500[W]} = 0.0061[°C/W]$$

$$R_{t1} = 0.06°C/W$$

4. *From Equation (4.48)*

$$\Delta x_B = R_{tB} k_B A$$

$$\text{Thickness of insulation} = 0.06\,[°C/W] \times 0.04[W/(m\,°C)] \times 18\,[m^2]$$
$$= 0.043 \text{ m} = 4.3 \text{ cm}$$

5. *An insulation with a thickness of 4.3 cm will ensure that heat loss from the wall will remain below 500 W. This thickness of insulation allows a 91% reduction in heat loss.*

4.4.3.2 *Composite Cylindrical Tube (in Series)*

Figure 4.18 shows a composite cylindrical tube made of two layers of materials, A and B. An example is a steel pipe covered with a layer of insulating material. The rate of heat transfer in this composite tube can be calculated as follows.

In Section 4.4.2 we found that rate of heat transfer through a single-wall cylinder is

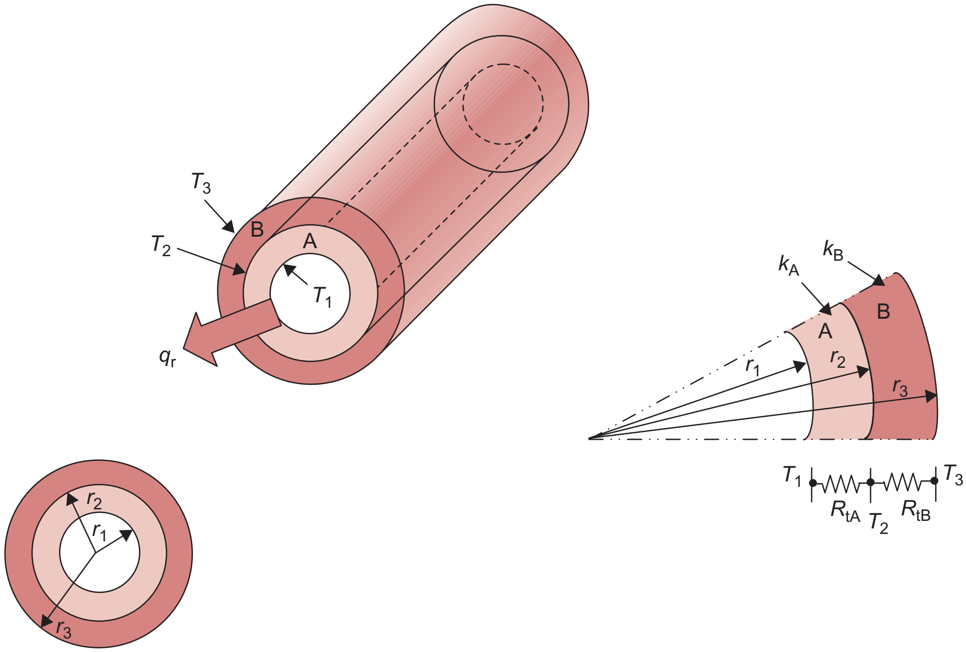
$$q_r = \frac{(T_i - T_o)}{\left[\dfrac{\ln(r_o/r_i)}{2\pi L k}\right]}$$

The rate of heat transfer through a composite cylinder using thermal resistances of the two layers is

$$q_r = \frac{(T_1 - T_3)}{R_{tA} + R_{tB}} \tag{4.49}$$

or, substituting the individual thermal resistance values,

$$q_r = \frac{(T_1 - T_3)}{\dfrac{\ln(r_2/r_1)}{2\pi L k_A} + \dfrac{\ln(r_3/r_2)}{2\pi L k_B}} \tag{4.50}$$



W

Figure 4.18 Conductive heat transfer in concentric cylindrical pipes, also shown with a thermal resistance circuit.

The preceding equation is useful in calculating the rate of heat transfer through a multilayered cylinder. Note that if there were three layers present between the two surfaces with temperatures T_1 and T_3, then we just add another thermal resistance term in the denominator.

Suppose we need to know the temperature at the interface between two layers, T_2, as shown in Figure 4.18. First, we calculate the steady-state rate of heat transfer using Equation (4.50), noting that under steady-state conditions, q_r has the same value through each layer of the composite wall. Then, we can use the following equation, which represents the thermal resistance between the known temperature, T_1, and the unknown temperature, T_2.

$$T_2 = T_1 - q\left(\frac{\ln(r_2/r_1)}{2\pi L k_A}\right) \tag{4.51}$$

This procedure to solve problems for unknown interfacial temperatures is illustrated in Example 4.9.

Example 4.9

A stainless-steel pipe (thermal conductivity = 17 W/[m °C]) is being used to convey heated oil (Fig. E4.9). The inside surface temperature is 130°C. The pipe is 2 cm thick with an inside diameter of 8 cm. The pipe is insulated with 0.04-m-thick insulation (thermal conductivity = 0.035 W/[m °C]). The outer insulation temperature is 25°C. Calculate the temperature of the interface between steel and insulation, assume steady-state conditions.

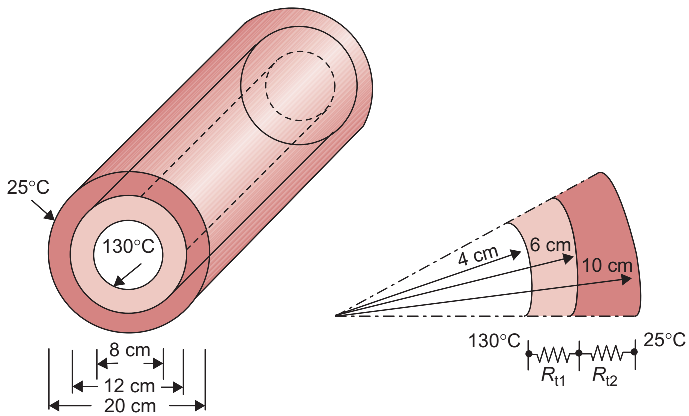


Figure E4.9 Heat transfer through a multilayered pipe.

Given

Thickness of pipe = 2 cm = 0.02 m
Inside diameter = 8 cm = 0.08 m
k_{steel} *= 17 W/(m °C)*
Thickness of insulation = 0.04 m
$k_{insulation}$ *= 0.035 W/(m °C)*
Inside pipe surface temperature = 130°C
Outside insulation surface temperature = 25°C
Pipe length = 1 m (assumed)

Approach

We will first calculate the two thermal resistances, in the pipe and the insulation. Then we will obtain the rate of heat transfer through the composite layer. Finally, we will use the thermal resistance of the pipe alone to determine the temperature at the interface between the pipe and insulation.

Solution

1. *Thermal resistance in the pipe layer is, from Equation (4.41),*

$$R_{t1} = \frac{\ln(0.06/0.04)}{2\pi \times 1[m] \times 17[W/(m\ °C)]}$$

$$= 0.0038\ °C/W$$

2. *Similarly, the thermal resistance in the insulation layer is,*

$$R_{t2} = \frac{\ln(0.1/0.06)}{2\pi \times 1\,[m] \times 0.035\,[W/(m\,°C)]}$$

$$= 2.3229\,°C/W$$

3. *Using Equation (4.49), the rate of heat transfer is*

$$q = \frac{(130 - 25)\,[°C]}{0.0038\,[°C/W] + 2.3229\,[°C/W]}$$

$$= 45.13\,W$$

4. *Using Equation (4.40)*

$$45.13\,[W] = \frac{(130 - T)\,[°C]}{0.0038\,[°C/W]}$$

$$T = 130\,[°C] - 0.171\,[°C]$$

$$T = 129.83°C$$

5. *The interfacial temperature is 129.8°C. This temperature is very close to the inside pipe temperature of 130°C, due to the high thermal conductivity of the steel pipe. The interfacial temperature between a hot surface and insulation must be known to ensure that the insulation will be able to withstand that temperature.*

Example 4.10

A stainless-steel pipe (thermal conductivity = 15 W/[mK]) is being used to transport heated oil at 125°C (Fig. E4.10). The inside temperature of the pipe is 120°C. The pipe has an inside diameter of 5 cm and is 1 cm thick. Insulation is necessary to keep the heat loss from the oil below 25 W/m length of the pipe. Due to space limitations, only 5-cm-thick insulation can be provided. The outside surface temperature of the insulation must be above 20°C (the dew point temperature of surrounding air) to avoid condensation of water on the surface of insulation. Calculate the thermal conductivity of insulation that will result in minimum heat loss while avoiding water condensation on its surface.

Given

Thermal conductivity of steel = 15 W/(m K)
Inside pipe surface temperature = 120°C
Inside diameter = 0.05 m
Pipe thickness = 0.01 m
Heat loss permitted in 1 m length of pipe = 25 W
Insulation thickness = 0.05 m
Outside surface temperature >20°C = 21°C (assumed)

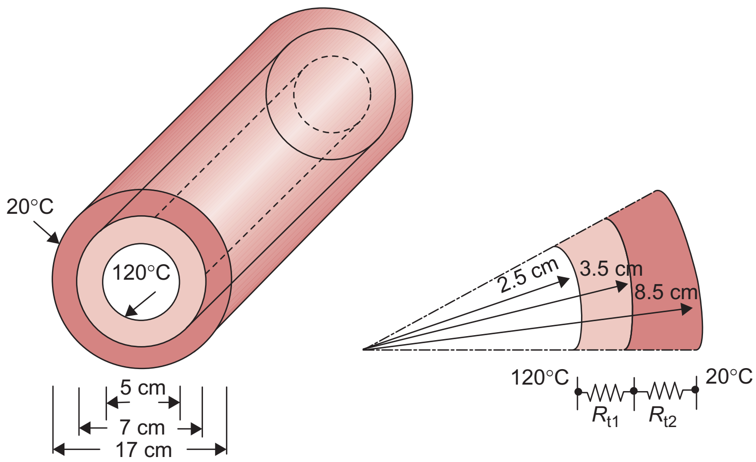


Figure E4.10 Heat transfer through a multilayered pipe.

Approach

We will first calculate the thermal resistance in the steel layer, and set up an equation for the thermal resistance in the insulation layer. Then we will substitute the thermal resistance values into Equation (4.50). The only unknown, thermal conductivity, k, will be then calculated.

Solution

1. *Thermal resistance in the steel layer is*

$$R_{t1} = \frac{\ln(3.5/2.5)}{2\pi \times 1\,[m] \times 15\,[W/(m\,°C)]} = 0.0036\,°C/W$$

2. *Thermal resistance in the insulation layer is*

$$R_{t2} = \frac{\ln(8.5/3.5)}{2\pi \times 1\,[m] \times k\,[W/(m\,°C)]} = \frac{0.1412[1/m]}{k\,[W/(m\,°C)]}$$

3. *Substituting the two thermal resistance values in Equation (4.50)*

$$25[W] = \frac{(120 - 21)[°C]}{0.0036[°C/W] + \dfrac{0.1412\,[1/m]}{k\,[W/(m\,°C)]}}$$

or,

$$k = 0.0357\,W/(m\,°C)$$

4. *An insulation with a thermal conductivity of 0.0357 W/(m °C) will ensure that no condensation will occur on its outer surface.*

4.4.4 Estimation of Convective Heat-Transfer Coefficient

In Section 4.3.1 on the conduction mode of heat transfer, we observed that any material undergoing conduction heating or cooling remains stationary. Conduction is the main mode of heat transfer within solids. Now we will consider heat transfer between a solid and a surrounding fluid, a mode of heat transfer called convection. In this case, the material experiencing heating or cooling (a fluid) also moves. The movement of fluid may be due to the natural buoyancy effects or caused by artificial means, such as a pump in the case of a liquid or a blower for air.

Determination of the rate of heat transfer due to convection is complicated because of the presence of fluid motion. In Chapter 2, we noted that a velocity profile develops when a fluid flows over a solid surface because of the viscous properties of the fluid material. The fluid next to the wall does not move but "sticks" to it, with an increasing velocity away from the wall. A boundary layer develops within the flowing fluid, with a pronounced influence of viscous properties of the fluid. This layer moves all the way to the center of a pipe, as was shown in Figure 2.14. The parabolic velocity profile under laminar flow conditions indicates that the drag caused by the sticky layer in contact with the solid surface influences velocity at the pipe center.

Similar to the velocity profile, a temperature profile develops in a fluid as it flows through a pipe, as shown in Figure 4.19. Suppose the temperature of the pipe surface is kept constant at T_s, and the fluid enters with a uniform temperature, T_i. A temperature profile develops because the fluid in contact with the pipe surface quickly reaches the wall temperature, thus setting up a temperature gradient as shown in the figure. A thermal boundary layer develops. At the end of the thermal entrance region, the boundary layer extends all the way to the pipe centerline.

Therefore, when heating or cooling a fluid as it flows through a pipe, two boundary layers develop—a hydrodynamic boundary

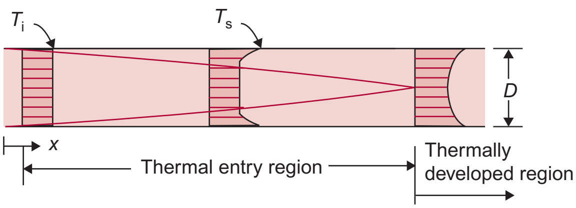


Figure 4.19 Thermal entry region in fluid flowing in a pipe.

layer and a thermal boundary layer. These boundary layers have a major influence on the rate of heat transfer between the pipe surface and the fluid. The mathematics involved in an analytical treatment of this subject is complicated and beyond the scope of this book. However, there is an equally useful procedure called the *empirical approach*, which is widely used to determine the rate of convective heat transfer. A drawback of the empirical approach is that it requires a large number of experiments to obtain the required data. We overcome this problem and keep the data analysis manageable by using dimensionless numbers. To formulate this approach, first we will identify and review the required dimensionless numbers: Reynolds number, N_{Re}, Nusselt number, N_{Nu}, and Prandtl number, N_{Pr}.

The Reynolds number was described in Section 2.3.2. It provides an indication of the inertial and viscous forces present in a fluid. The Reynolds number is calculated using Equation (2.20).

The second required dimensionless number for our data analysis is Nusselt number—the dimensionless form of convective heat transfer coefficient, h. Consider a fluid layer of thickness l, as shown in Figure 4.20. The temperature difference between the top and bottom of the layer is ΔT. If the fluid is stationary, then the rate of heat transfer will be due to conduction, and the rate of heat transfer will be

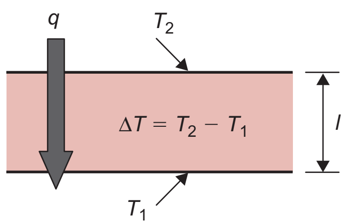
$$q_{\text{conduction}} = -kA\frac{\Delta T}{l} \tag{4.52}$$

However, if the fluid layer is moving, then the heat transfer will be due to convection, and the rate of heat transfer using Newton's law of cooling will be

$$q_{\text{convection}} = hA\Delta T \tag{4.53}$$

Dividing Equation (4.53) by (4.52), we get

$$\frac{q_{\text{convection}}}{q_{\text{conduction}}} = \frac{hA\Delta T}{kA\Delta T/l} = \frac{hl}{k} \equiv N_{Nu} \tag{4.54}$$



■ **Figure 4.20** Heat transfer through a fluid layer.

Replacing thickness l with a more general term for dimension, the characteristic dimension d_c, we get

$$N_{Nu} \equiv \frac{hd_c}{k} \tag{4.55}$$

Nusselt number may be viewed as an enhancement in the rate of heat transfer caused by convection over the conduction mode. Therefore, if $N_{Nu} = 1$, then there is no improvement in the rate of heat transfer due to convection. However, if $N_{Nu} = 5$, the rate of convective heat transfer due to fluid motion is five times the rate of heat transfer if the fluid in contact with the solid surface is stagnant. The fact that by blowing air over a hot surface we can cool it faster is due to increased Nusselt number and consequently to an increased rate of heat transfer.

The third required dimensionless number for the empirical approach to determine convective heat transfer is Prandtl number, N_{Pr}, which describes the thickness of the hydrodynamic boundary layer compared with the thermal boundary layer. It is the ratio between the molecular diffusivity of momentum to the molecular diffusivity of heat. Or,

$$N_{Pr} = \frac{\text{molecular diffusivity of momentum}}{\text{molecular diffusivity of heat}} \tag{4.56}$$

or,

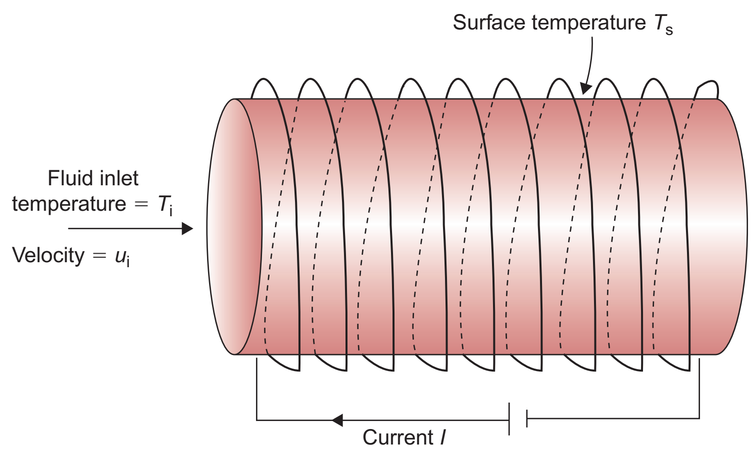
$$N_{Pr} = \frac{\text{kinematic viscosity}}{\text{thermal diffusivity}} = \frac{\nu}{\alpha} \tag{4.57}$$

Substituting Equations (2.11) and (4.12) in Equation (4.57),

$$N_{Pr} = \frac{\mu c_p}{k} \tag{4.58}$$

If $N_{Pr} = 1$, then the thickness of the hydrodynamic and thermal boundary layers will be exactly the same. On the other hand, if $N_{Pr} \ll 1$, the molecular diffusivity of heat will be much larger than that of momentum. Therefore, the heat will dissipate much faster, as in the case of a liquid metal flowing in a pipe. For gases, N_{Pr} is about 0.7, and for water it is around 10.

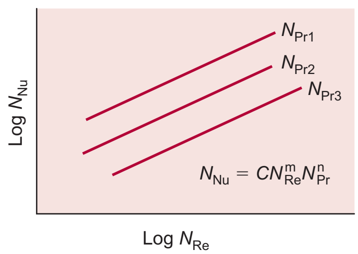
With a basic understanding of these three dimensionless numbers, we will now plan the following experiment to determine convective rate of heat transfer. Assume that a fluid is flowing in a heated pipe. We are interested in determining convective rate of heat transfer from the inside surface of the heated pipe into the fluid flowing inside the pipe, as shown in Figure 4.21. We carry out this experiment by pumping a fluid such as water, entering at a velocity of u_i at a temperature of T_i and flowing parallel to the inside surface of the pipe. The pipe



■ **Figure 4.21** Heating of fluid with electrically heated pipe surface.

is heated using an electrical heater so that the inside pipe surface is maintained at temperature T_s, which is higher than the inlet fluid temperature, T_i. We measure the electric current, I, and electrical resistance, R_E, and calculate the product of the two to determine the rate of heat transfer, q. The pipe is well insulated so that all the electrically generated heat transfers into the fluid. Thus, we can experimentally determine values of q, A, T_i, u_i, and T_s. Using Equation (4.53), we can calculate the convective heat transfer coefficient, h.

If we repeat this experiment with a different diameter pipe or different temperature of pipe surface, a new value of h will be obtained. It should become clear that we can perform a series of experiments to obtain h values that are a function of the operating variables, q, A, u_i, T_i, and T_s. The disadvantage of this experiment is that a large amount of experimental data are generated, and organizing these data for meaningful applications is a daunting task. However, the data analysis can be greatly simplified if we combine various properties and operating variables into the three dimensionless numbers, N_{Re}, N_{Nu}, and N_{Pr}, which will accommodate all the properties and variables that are important to our experiment.



■ **Figure 4.22** A plot of Nusselt and Reynolds numbers on a log-log scale.

Thus, for each experimental set, we calculate the respective dimensionless numbers and, using log-log scale, plot the Nusselt number as a function of the Reynolds number for different values of the Prandtl number. Figure 4.22 shows a typical plot. It has been experimentally determined that for a given fluid with a fixed Prandtl number, straight line plots are obtained on the log-log scale, as shown in Figure 4.22.

This type of graphical relationship may be conveniently expressed with an equation as

$$N_{\mathrm{Nu}} = C N_{\mathrm{Re}}^{m} N_{\mathrm{Pr}}^{n} \tag{4.59}$$

where C, m, and n are coefficients.

By substituting the experimentally obtained coefficients in Equation (4.59), we obtain *empirical correlations* specific for a given condition. Several researchers have determined these empirical correlations for a variety of operating conditions, such as fluid flow inside a pipe, over a pipe, or over a sphere. Different correlations are obtained, depending on whether the flow is laminar or turbulent.

A suggested methodology to solve problems requiring the calculation of convective heat transfer coefficients using empirical correlations is as follows:

1. *Identify flow geometry*. The first step in a calculation involving convection heat transfer is to clearly identify the geometrical shape of the solid surface in contact with the fluid and its dimensions. For example, is it a pipe, sphere, rectangular duct, or a rectangular plate? Is the fluid flowing inside a pipe or over the outside surface?

2. *Identify the fluid and determine its properties*. The second step is to identify the type of fluid. Is it water, air, or a liquid food? Determine the average fluid temperature far away from the solid surface, T_{∞}. In some cases the average inlet and outlet temperatures may be different, for example, in a heat exchanger; in that case calculate the average fluid temperature as follows:

$$T_{\infty} = \frac{T_{\mathrm{i}} + T_{\mathrm{e}}}{2} \tag{4.60}$$

   where T_{i} is the average inlet fluid temperature and T_{e} is the average exit fluid temperature. Use the average fluid temperature, T_{∞}, to obtain physical and thermal properties of the fluid, such as viscosity, density, and thermal conductivity, from appropriate tables (such as Table A.4.1 for water, Table A.4.4 for air), paying careful attention to the units of each property.

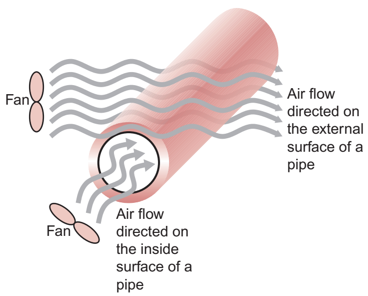
3. *Calculate the Reynolds number*. Using the velocity of the fluid, fluid properties and the characteristic dimension of the object

in contact with the fluid, calculate the Reynolds number. The Reynolds number is necessary to determine whether the flow is laminar, transitional, or turbulent. This information is required to select an appropriate empirical correlation.

4. *Select an appropriate empirical correlation*. Using the information from steps (1) and (3), select an empirical correlation of the form given in Equation (4.59) for the conditions and geometry of the object that resembles the one being investigated (as presented later in this section). For example, if the given problem involves turbulent water flow in a pipe, select the correlation given in Equation (4.67). Using the selected correlation, calculate Nusselt number and finally the convective heat transfer coefficient.

The convective heat-transfer coefficient h is predicted from empirical correlations. The coefficient is influenced by such parameters as type and velocity of the fluid, physical properties of the fluid, temperature difference, and geometrical shape of the physical system under consideration.

The empirical correlations useful in predicting h are presented in the following sections for both forced and free convection. We will discuss selected physical systems that are most commonly encountered in convective heat transfer in food processing. For other situations refer to handbooks such as Rotstein et al. (1997) or Heldman and Lund (1992). All correlations apply to Newtonian fluids only. For expressions for non-Newtonian fluids, the textbook by Heldman and Singh (1981) is recommended.



W

Figure 4.23 Forced convective heat transfer from a pipe with flow inside and outside the pipe.

4.4.4.1 *Forced Convection*

In forced convection, a fluid is forced to move over a solid surface by external mechanical means, such as an electric fan, pump, or a stirrer (Fig. 4.23). The general correlation between the dimensionless numbers is

$$N_{Nu} = \Phi(N_{Re}, N_{Pr}) \tag{4.61}$$

where N_{Nu} is Nusselt number $= hd_c/k$; h is convective heat-transfer coefficient (W/[m^2 °C]); d_c is the characteristic dimension (m); k is thermal conductivity of fluid (W/[m °C]); N_{Re} is Reynolds number $= \rho \bar{u} d_c/\mu$; ρ is density of fluid (kg/m^3); $\bar{u}$ is velocity of fluid (m/s); μ is viscosity (Pa s); N_{Pr} is Prandtl number $= \mu c_p/k$; c_p is specific heat (kJ/[kg °C]); and Φ stands for "function of".

Laminar flow in pipes

1. Fully developed conditions with constant surface temperature of the pipe:

$$N_{\mathrm{Nu}} = 3.66 \tag{4.62}$$

where thermal conductivity of the fluid is obtained at average fluid temperature, T_∞, and d_c is the inside diameter of the pipe.

2. Fully developed conditions with uniform surface heat flux:

$$N_{\mathrm{Nu}} = 4.36 \tag{4.63}$$

where thermal conductivity of the fluid is obtained at average fluid temperature, T_∞, and d_c is the inside diameter of the pipe.

3. For both entry region and fully developed flow conditions:

$$N_{\mathrm{Nu}} = 1.86\left(N_{\mathrm{Re}} \times N_{\mathrm{Pr}} \times \frac{d_c}{L}\right)^{0.33}\left(\frac{\mu_b}{\mu_w}\right)^{0.14} \tag{4.64}$$

where L is the length of pipe (m); characteristic dimension, d_c, is the inside diameter of the pipe; all physical properties are evaluated at the average fluid temperature, T_∞, except μ_w, which is evaluated at the surface temperature of the wall.

Transition flow in pipes

For Reynolds numbers between 2100 and 10,000,

$$N_{\mathrm{Nu}} = \frac{(f/8)(N_{\mathrm{Re}} - 1000)N_{\mathrm{Pr}}}{1 + 12.7(f/8)^{1/2}(N_{\mathrm{Pr}}^{2/3} - 1)} \tag{4.65}$$

where all fluid properties are evaluated at the average fluid temperature, T_∞, d_c is the inside diameter of the pipe, and the friction factor, f, is obtained for smooth pipes using the following expression:

$$f = \frac{1}{(0.790 \ln N_{\mathrm{Re}} - 1.64)^2} \tag{4.66}$$

Turbulent flow in pipes

The following equation may be used for Reynolds numbers greater than 10,000:

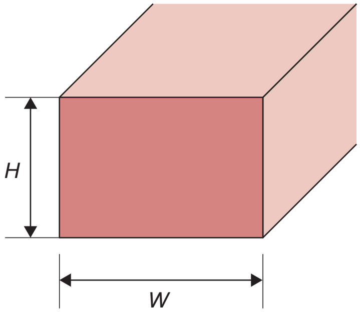
$$N_{\mathrm{Nu}} = 0.023 N_{\mathrm{Re}}^{0.8} \times N_{\mathrm{Pr}}^{0.33} \times \left(\frac{\mu_b}{\mu_w}\right)^{0.14} \tag{4.67}$$

Fluid properties are evaluated at the average film temperature, T_∞, except μ_w, which is evaluated at the wall temperature; d_c is the inside diameter of the pipe. Equation (4.67) is valid both for constant surface temperature and uniform heat flux conditions.

Convection in noncircular ducts

For noncircular ducts, an equivalent diameter, D_e, is used for the characteristic dimension:

$$D_e = \frac{4 \times \text{free area}}{\text{wetted perimeter}} \tag{4.68}$$



■ **Figure 4.24** Cross-section of a rectangular duct.

Figure 4.24 shows a rectangular duct with sides of length W and H. The equivalent diameter in this case will be equal to 2 WH/(W + H).

Flow past immersed objects

In several applications, the fluid may flow past immersed objects. For these cases, the heat transfer depends on the geometrical shape of the object, relative position of the object, proximity of other objects, flow rate, and fluid properties.

For a flow past a single sphere, when the single sphere may be heated or cooled, the following equation will apply:

$$N_{Nu} = 2 + 0.60 N_{Re}^{0.5} \times N_{Pr}^{1/3} \qquad \text{for} \begin{cases} 1 < N_{Re} < 70{,}000 \\ 0.6 < N_{Pr} < 400 \end{cases} \tag{4.69}$$

where the characteristic dimension, d_c, is the outside diameter of the sphere. The fluid properties are evaluated at the film temperature T_f where

$$T_f = \frac{T_s + T_\infty}{2}$$

For heat transfer in flow past other immersed objects, such as cylinders and plates, correlations are available in Perry and Chilton (1973).

Example 4.11

Water flowing at a rate of 0.02 kg/s is heated from 20 to 60°C in a horizontal pipe (inside diameter = 2.5 cm). The inside pipe surface temperature is 90°C (Fig. E4.11). Estimate the convective heat-transfer coefficient if the pipe is 1 m long.

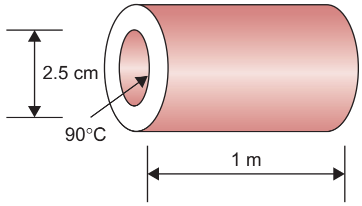


Figure E4.11 Convective heat transfer inside a pipe.

Given

Water flow rate = 0.02 kg/s
Inlet temperature = 20°C
Exit temperature = 60°C
Inside diameter = 2.5 cm = 0.025 m
Inside pipe surface temperature = 90°C
Length of pipe = 1 m

Approach

Since water is flowing due to some external means, the problem indicates forced convective heat transfer. We will first determine if the flow is laminar by calculating the Reynolds number. If the Reynolds number is less than 2100, we will use Equation (4.64) to calculate the Nusselt number. From the Nusselt number we will calculate the h value.

Solution

1. *Physical properties of water are needed to calculate the Reynolds number. All physical properties except u_w must be evaluated at average bulk fluid temperature, (20 + 60)/2 = 40°C. From Table A.4.1 at 40°C,*
   Density $\rho = 992.2$ kg/m^3
   Specific heat $c_p = 4.175$ kJ/(kg °C)
   Thermal conductivity $k = 0.633$ W/(m °C)
   Viscosity (absolute) $\mu = 658.026 \times 10^{-6}$ Pa s
   Prandtl number $N_{Pr} = 4.3$

   Thus,

$$N_{Re} = \frac{\rho \bar{u} D}{\mu} = \frac{4\dot{m}}{\pi \mu D}$$

$$= \frac{(4)(0.02\ \text{kg/s})}{(\pi)(658.026 \times 10^{-6}\ \text{P a s})(0.025\ \text{m})}$$

$$= 1547.9$$

   Note that 1 Pa = 1 kg/(m s^2). Since the Reynolds number is less than 2100, the flow is laminar.

2. *We select Equation (4.64), and using $\mu_w = 308.909 \times 10^{-6}$ Pa s at 90°C,*

$$N_{Nu} = 1.86(1547.9 \times 4.3 \times 0.025)^{0.33} \left(\frac{658.016 \times 10^{-6}}{308.909 \times 10^{-6}} \right)^{0.14}$$

$$= 11.2$$

3. *The convective heat-transfer coefficient can be obtained from the Nusselt number.*

$$h = \frac{N_{Nu}k}{D} = \frac{(11.2)(0.633\ W/[m\ °C])}{(0.025\ m)}$$

$$= 284\ W/(m^2\ °C)$$

4. *The convective heat-transfer coefficient is estimated to be 284 W/(m^2 °C).*

Example 4.12

If the rate of water flow in Example 4.11 is raised to 0.2 kg/s from 0.02 kg/s while all other conditions are kept the same, calculate the new convective heat-transfer coefficient.

Given

See Example 4.11

New mass flow rate of water = 0.2 kg/s

Approach

We will calculate the Reynolds number to find whether the flow is turbulent. If the flow is turbulent, we will use Equation (4.67) to compute the Nusselt number. The surface heat-transfer coefficient will be computed from the Nusselt number.

Solution

1. *First, we compute the Reynolds number using some of the properties obtained in Example 4.11.*

$$N_{Re} = \frac{(4)(0.2\ kg/s)}{(\pi)(658.026 \times 10^{-6}\ Pa\ s)(0.025\ m)} = 15{,}479$$

Thus, flow is turbulent.

2. *For turbulent flow, we select Equation (4.67).*

$$N_{Nu} = (0.023)(15479)^{0.8}(4.3)^{0.33}\left(\frac{658.026 \times 10^{-6}}{308.909 \times 10^{-6}}\right)^{0.14}$$

$$= 93$$

3. *The convective heat transfer can be computed as*

$$h = \frac{N_{Nu}k}{D} = \frac{(93)(0.633\ W/[m\ °C])}{(0.025\ m)}$$

$$= 2355\ W/(m^2\ °C)$$

4. *The convective heat-transfer coefficient for turbulent flow is estimated to be 2355 W/(m² °C). This value is more than eight times higher than the value of h for laminar flow calculated in Example 4.11.*

Example 4.13

What is the expected percent increase in convective heat-transfer coefficient if the velocity of a fluid is doubled while all other parameters are kept the same for turbulent flow in a pipe?

Approach

We will use Equation (4.67) to solve this problem.

Solution

1. *For turbulent flow in a pipe*

$$N_{Nu} = 0.023 N_{\mathrm{Re}}^{0.8} \times N_{\mathrm{Pr}}^{0.33} \times \left(\frac{\mu_b}{\mu_w}\right)^{0.14}$$

We can rewrite this equation as

$$N_{Nu_1} = f(\bar{u}_1)^{0.8}$$

$$N_{Nu_2} = f(\bar{u}_2)^{0.8}$$

$$\frac{N_{Nu_2}}{N_{Nu_1}} = \left(\frac{\bar{u}_2}{\bar{u}_1}\right)^{0.8}$$

2. *Since* $\bar{u}_2 = 2\bar{u}_1$,

$$\frac{N_{Nu_2}}{N_{Nu_1}} = (2)^{0.8} = 1.74$$

$$N_{Nu_2} = 1.74 N_{Nu_1}$$

3. *This expression implies that* $h_2 = 1.74\, h_1$. *Thus,*

$$\%Increase = \frac{1.74h_1 - h_1}{h_1} \times 100 = 74\%$$

4. *As expected, velocity has a considerable effect on the convective heat-transfer coefficient.*

Example 4.14

Calculate convective heat-transfer coefficient when air at 90°C is passed through a deep bed of green peas. Assume surface temperature of a pea to be 30°C. The diameter of each pea is 0.5 cm. The velocity of air through the bed is 0.3 m/s.

Given

Diameter of pea = 0.005 m
Temperature of air = 90°C
Temperature of a pea = 30°C
Velocity of air = 0.3 m/s

Approach

Since the air flows around a spherically immersed object (green pea), we will estimate N_{Nu} from Equation (4.69). The Nusselt number will give us the value for h.

Solution

1. *The properties of air are evaluated at T_f, where*

$$T_f = \frac{T_s + T_\infty}{2} = \frac{30+90}{2} = 60°C$$

From Table A.4.4,

$$\rho = 1.025\ kg/m^3$$
$$c_p = 1.017\ kJ/(kg\ °C)$$
$$k = 0.0279\ W/(m\ °C)$$
$$\mu = 19.907 \times 10^{-6}\ Pa\ s$$
$$N_{Pr} = 0.71$$

2. *The Reynolds number is computed as*

$$N_{Re} = \frac{(1.025\ kg/m^3)(0.3\ m/s)(0.005\ m)}{(19.907 \times 10^{-6}\ Pa\ s)}$$
$$= 77.2$$

3. *From Equation (4.69),*

$$N_{Nu} = 2 + 0.6(77.2)^{0.5}(0.71)^{0.33}$$
$$= 6.71$$

4. *Thus*

$$h = \frac{6.71(0.0279\,W/[m\,°C])}{(0.005\,m)} = 37\,W/(m^2\,°C)$$

5. *The convective heat-transfer coefficient is 37 W/(m^2 °C).*

4.4.4.2 *Free Convection*

Free convection occurs because of density differences in fluids as they come into contact with a heated surface (Fig. 4.25). The low density of fluid at a higher temperature causes buoyancy forces, and as a result, heated fluid moves upward and colder fluid takes its place.

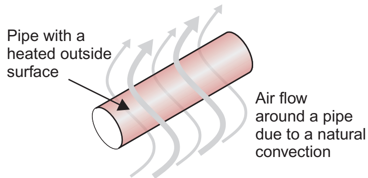


Figure 4.25 Heat transfer from the outside of a heated pipe due to natural convection.

Empirical expressions useful in predicting convective heat-transfer coefficients are of the following form:

$$N_{Nu} = \frac{hd_c}{k} = a(N_{Ra})^m \tag{4.70}$$

where a and m are constants; N_{Ra}, is the Rayleigh number. Rayleigh number is a product of two dimensionless numbers, Grashof number and Prandtl number.

$$N_{Ra} = N_{Gr} \times N_{Pr} \tag{4.71}$$

The Grashof number, N_{Gr}, is defined as follows:

$$N_{Gr} = \frac{d_c^3 \rho^2 g \beta \Delta T}{\mu^2} \tag{4.72}$$

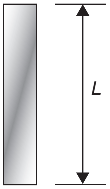
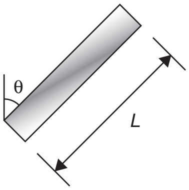
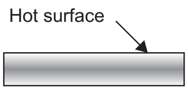
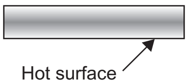
where d_c is characteristic dimension (m); ρ is density (kg/m^3); g is acceleration due to gravity (9.80665 m/s^2); β is coefficient of volumetric expansion (K^{-1}); ΔT is temperature difference between wall and the surrounding bulk (°C); and μ is viscosity (Pa s).

A Grashof number is a ratio between the buoyancy forces and viscous forces. Similar to the Reynolds number, the Grashof number is useful for determining whether a flow over an object is laminar or turbulent. For example, a Grashof number greater than 10^9 for fluid flow over vertical plates signifies a turbulent flow.

In the case of heat transfer due to free convection, physical properties are evaluated at the film temperature, $T_f = (T_s + T_\infty)/2$.

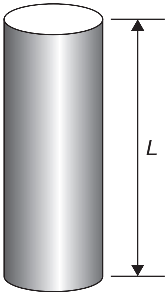
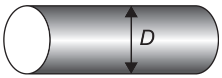
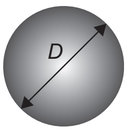
Table 4.2 gives various constants that may be used in Equation (4.70) for natural convection from vertical plates and cylinders, and from horizontal cylinders and plates.

Table 4.2 Coefficients for Equation (4.70) for Free Convection

Geometry	Characteristic Length	Range of N_{Ra}	a	m	Equation
Vertical plate 	L	10^4–10^9 10^9–10^{13}	0.59 0.1	0.25 0.333	$N_{Nu} = a(N_{Ra})^m$
Inclined plate 	L				Use same equations as vertical plate, replace g by $g \cos\theta$ for $N_{Ra} < 10^9$
Horizontal plate Surface area = A Perimeter = p (a) Upper surface of a hot plate (or lower surface of a cold plate) 	A/p	10^4–10^7 10^7–10^{11}	0.54 0.15	0.25 0.33	$N_{Nu} = a(N_{Ra})^m$
Horizontal plate Surface area = A Perimeter = p (b) Lower surface of a hot plate (or upper surface of a cold plate) 	A/p	10^5–10^{11}	0.27	0.25	$N_{Nu} = a(N_{Ra})^m$

(*Continued*)

Table 4.2 (Continued)

Geometry	Characteristic Length	Range of N_{Ra}	a	m	Equation
Vertical cylinder 	L				A vertical cylinder can be treated as a vertical plate when $D \geq \frac{35L}{N_{Gr}^{0.25}}$
Horizontal cylinder 	D	10^{-5}–10^{-12}			$N_{Nu} = \left\{ 0.6 + \frac{0.387 N_{Ra}^{1/6}}{\left[1 + \left(\frac{0.559}{N_{Pr}}\right)^{9/16}\right]^{8/27}} \right\}^2$
Sphere 	$\frac{1}{2}\pi D$	$N_{Ra} \leq 10^{11}$ ($N_{Pr} \geq 0.7$)			$N_{Nu} = 2 + \frac{0.589 N_{Ra}^{1/4}}{\left[1 + \left(\frac{0.469}{N_{Pr}}\right)^{9/16}\right]^{4/9}}$

Example 4.15

Estimate the convective heat-transfer coefficient for convective heat loss from a horizontal 10 cm diameter steam pipe. The surface temperature of the uninsulated pipe is 130°C, and the air temperature is 30°C (Fig. E4.12).

Given

Diameter of pipe = 10 cm = 0.1 m
Pipe surface temperature T_w = 130°C
Ambient temperature T_∞ = 30°C

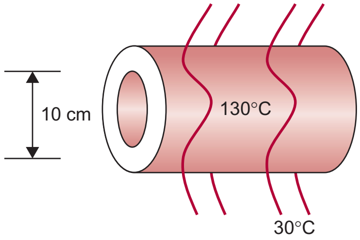


Figure E4.12 Convective heat transfer from a horizontal pipe.

Approach

Since no mechanical means of moving air are indicated, heat loss from the horizontal pipe is by free convection. After finding the property values of air at film temperature, we will calculate the Grashof number. The product of the Grashof number and Prandtl number will allow determination of a and m parameters from Table 4.2; these parameters will be used in Equation (4.72). We will then compute the surface heat-transfer coefficient from the Nusselt number.

Solution

1. *Since no mechanical means of moving the air are indicated, heat loss is by free convection.*
2. *The film temperature is obtained as*

$$T_f = \frac{T_s + T_\infty}{2} = \frac{130 + 30}{2} = 80°C$$

3. *The properties of air at 80°C are obtained from Table A.4.4.*

$$\rho = 0.968\ kg/m^3$$
$$\beta = 2.83 \times 10^{-3}\ K^{-1}$$
$$c_p = 1.019\ kJ/(kg\ °C)$$
$$k = 0.0293\ W/(m\ °C)$$
$$\mu = 20.79 \times 10^{-6}\ N\ s/m^2$$
$$N_{Pr} = 0.71$$
$$g = 9.81\ m/s^2$$

4. *We calculate Rayleigh number, N_{Ra}, the product of N_{Gr} and N_{Pr}. The characteristic dimension is the outside diameter of the pipe.*

$$N_{Gr} = \frac{d_c^3 \rho^2 g \beta \Delta T}{\mu^2}$$
$$= \frac{(0.1\,m)^3 (0.968\,kg/m^3)^2 (9.81\,m/s^2)(2.83 \times 10^{-3}\,K^{-1})(130°C - 30°C)}{(20.79 \times 10{-}6\,N\,s/m^2)^2}$$
$$= 6.019 \times 10^6$$

(Note that 1 N = kg m/s^2.) Thus,

$$N_{Gr} \times N_{Pr} = (6.019 \times 10^6)(0.71) = 4.27 \times 10^6$$

5. *From Table 4.2, for horizontal cylinder*

$$N_{Nu} = \left(0.6 + \frac{0.387(4.27 \times 10^6)^{1/6}}{\left(1 + \left(\frac{0.559}{0.71}\right)^{9/16}\right)^{8/27}} \right)^2$$

6. $N_{Nu} = 22$

7. *Thus*

$$h = \frac{(22)(0.0293\ \mathrm{W/[m\ °C]})}{(0.1\ \mathrm{m})} = 6.5\ \mathrm{W/(m^2\ °C)}$$

4.4.4.3 *Thermal Resistance in Convective Heat Transfer*

A thermal resistance term for convective heat transfer may be defined in a similar manner as in conductive heat transfer (Section 4.4). From Equation (4.19), we know that

$$q = hA(T_s - T_\infty) \tag{4.73}$$

or, rearranging terms in Equation (4.73),

$$q = \frac{T_s - T_\infty}{\left(\frac{1}{hA}\right)} \tag{4.74}$$

where the thermal resistance due to convection $(R_t)_{convection}$ is

$$(R_t)_{convection} = \frac{1}{hA} \tag{4.75}$$

In problems involving conduction and convection heat transfer in series, along the path of heat transfer, the thermal resistance due to convection is added to the thermal resistance due to conduction to obtain the total thermal resistance. We will discuss this further in the context of overall heat transfer involving both conduction and convection heat transfer.

4.4.5 Estimation of Overall Heat-Transfer Coefficient

In many heating/cooling applications, conductive and convective heat transfer may occur simultaneously. An example shown in Figure 4.26 involves heat transfer in a pipe that carries a fluid at a temperature greater than the temperature of the environment surrounding the outside of the pipe. In this case, heat must first transfer from the inside fluid by forced convection to the inside surface of the pipe, then by conduction through the pipe wall material, and finally by free convection from the outer pipe surface to the surrounding environment. Thus, heat transfer is through three layers in a series.

Using the approach of thermal resistance values, we can write:

$$q = \frac{T_i - T_\infty}{R_t} \tag{4.76}$$

where R_t is a combination of the thermal resistances in the inside convective layer, the conductive layer in the pipe material, and the outside convective layer, or

$$R_t = (R_t)_{\text{inside convection}} + (R_t)_{\text{conduction}} + (R_t)_{\text{outiside convection}} \tag{4.77}$$

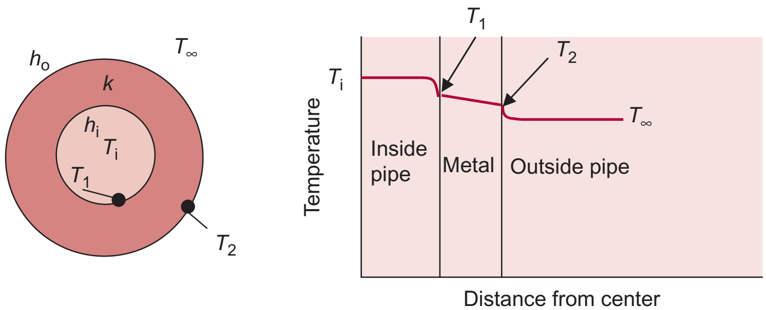
where

$$(R_t)_{\text{inside convection}} = \frac{1}{h_i A_i} \tag{4.78}$$

where h_i is the inside convective heat transfer coefficient, and A_i is the inside surface area of the pipe.

Resistance to heat transfer in the pipe wall is

$$(R_t)_{\text{conduction}} = \frac{\ln\left(\dfrac{r_o}{r_i}\right)}{2\pi k L} \tag{4.79}$$



■ **Figure 4.26** Combined conductive and convective heat transfer.

where k is the thermal conductivity of the pipe material (W/[m K]), r_i is the inside radius (m), and r_o is the outside radius (m). Resistance to heat transfer due to convection at the outside pipe surface is

$$(R_t)_{\text{outiside convection}} = \frac{1}{h_o A_o} \tag{4.80}$$

where h_o is the convective heat transfer coefficient at the outside surface of the pipe (W/[m^2 K]), and A_o is the outside surface area of the pipe. Substituting Equations (4.78), (4.79), and (4.80) in Equation (4.76), we obtain

$$q = \frac{T_i - T_\infty}{\dfrac{1}{h_i A_i} + \dfrac{\ln(r_o/r_i)}{2\pi L k} + \dfrac{1}{h_o A_o}} \tag{4.81}$$

We can also write an expression for the overall heat transfer for this example as

$$q = U_i A_i (T_i - T_\infty) \tag{4.82}$$

where A_i is the inside area of the pipe, and U_i is the overall heat-transfer coefficient based on the inside area of the pipe. From Equation (4.82),

$$q = \frac{T_i - T_\infty}{\left(\dfrac{1}{U_i A_i}\right)} \tag{4.83}$$

From Equations (4.83) and (4.81) we obtain

$$\frac{1}{U_i A_i} = \frac{1}{h_i A_i} + \frac{\ln\left(\dfrac{r_o}{r_i}\right)}{2\pi L k} + \frac{1}{h_o A_o} \tag{4.84}$$

Equation (4.84) is used to calculate the overall heat-transfer coefficient. The selection of area over which to calculate the overall heat transfer is quite arbitrary. For example, if U_o is selected as the overall heat-transfer coefficient based on outside area of the pipe, then Equation (4.84) is written as

$$\frac{1}{U_o A_o} = \frac{1}{h_i A_i} + \frac{\ln\left(\dfrac{r_o}{r_i}\right)}{2\pi L k} + \frac{1}{h_o A_o} \tag{4.85}$$

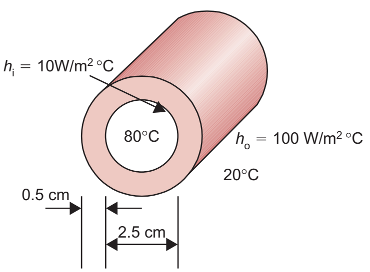
and Equation (4.82) is modified to

$$q = U_o A_o (T_i - T_\infty) \tag{4.86}$$

Both Equations (4.82) and (4.86) yield the same value of the rate of heat transfer, q. This is shown in Example 4.16.

Example 4.16

A 2.5-cm inside diameter pipe is being used to convey a liquid food at 80°C (Fig. E4.13). The inside convective heat transfer coefficient is 10 W/(m^2 °C). The pipe (0.5 cm thick) is made of steel (thermal conductivity = 43 W/[m °C]). The outside ambient temperature is 20°C. The outside convective heat-transfer coefficient is 100 W/(m^2 °C). Calculate the overall heat transfer coefficient and the heat loss from 1 m length of the pipe.



■ **Figure E4.13** Overall heat transfer in a pipe.

Given

Inside diameter of pipe = 0.025 m
Bulk temperature of liquid food = 80°C
Inside convective heat-transfer coefficient = 10 W/(m^2 °C)
Outside convective heat-transfer coefficient = 100 W/(m^2 °C)
k_{steel} *= 43 W/(m °C)*
Outside ambient temperature = 20°C

Approach

The overall heat-transfer coefficient can be computed by using a basis of either the inside area of the pipe or the outside area of the pipe. We will use Equation (4.84) to find U_i and then use a modification of Equation (4.84) to find U_o. We will prove that the computed rate of heat flow will remain the same regardless of whether U_i or U_o is selected.

Solution

1. *Calculate the overall heat-transfer coefficient based on inside area using Equation (4.84):*

$$\frac{1}{U_i A_i} = \frac{1}{h_i A_i} + \frac{\ln\left(\frac{r_o}{r_i}\right)}{2\pi L k} + \frac{1}{h_o A_o}$$

2. *By canceling area terms and noting that $A_i = 2\pi r_i L$,*

$$\frac{1}{U_i} = \frac{1}{h_i} + \frac{r_i \ln\left(\frac{r_o}{r_i}\right)}{k} + \frac{r}{h_o r_o}$$

3. *Substituting,*

$$\frac{1}{U_i} = \frac{1}{10\,[W/(m^2\,°C)]} + \frac{0.0125\,[m] \times \ln\left(\frac{0.0175}{0.0125}\right)\left[\frac{m}{m}\right]}{43\,[W/(m\,°C)]}$$

$$+ \frac{0.0125\,[m]}{100\,[W/(m^2\,°C)] \times 0.0175\,[m]}$$

$$= 0.1 + 0.0001 + 0.00714 = 0.10724\ m^2\,°C/W$$

Thus, $U_i = 9.32\ W/(m^2\,°C)$.

4. *Heat loss*

$$q = U_iA_i(80 - 20)$$

$$= 9.32\,[W/(m^2\,°C)] \times 2\pi \times 1\,[m] \times 0.0125\,[m] \times 60\,[°C]$$

$$= 43.9\ W$$

5. *Overall heat transfer coefficient based on outside area may be computed as*

$$\frac{1}{U_oA_o} = \frac{1}{h_iA_i} + \frac{\ln\left(\frac{r_o}{r_i}\right)}{2\pi kL} + \frac{1}{h_oA_o}$$

6. *By canceling area terms and noting that* $A_o = 2\pi r_oL$,

$$\frac{1}{U_o} = \frac{r_o}{h_iA_i} + \frac{r_o\ln\left(\frac{r_o}{r_i}\right)}{k} + \frac{1}{h_o}$$

Substituting,

$$\frac{1}{U_o} = \frac{0.0175\,[m]}{10\,[W/(m^2\,°C)] \times 0.0125\,[m]}$$

$$+ \frac{0.0175\,[m] \times \ln\left(\frac{0.0175}{0.0125}\right)\left[\frac{m}{m}\right]}{43\,[W/(m\,°C)]}$$

$$+ \frac{1}{100\,[W/(m^2\,°C)]}$$

$$= 0.14 + 0.00014 + 0.01$$

$$= 0.1501\ m^2\,°C/W$$

$$U_o = 6.66\ W/(m^2\,°C)$$

7. *Heat loss*

$$q = U_o A_o (80 - 20)$$
$$= 6.66\,[W/m^2\,°C] \times 2\pi \times 0.0175\,[m] \times 1\,[m] \times 60\,[°C]$$
$$= 43.9\ W$$

8. *As expected, the rate of heat loss remains the same regardless of which area was selected for computing overall heat-transfer coefficient.*
9. *It should be noted from steps (3) and (6) that the resistance offered by the metal wall is considerably smaller than the resistance offered in the convective layers.*

4.4.6 Fouling of Heat Transfer Surfaces

In heating equipment, when a liquid food comes into contact with a heated surface, some of its components may deposit on the hot surface, causing an increase in the resistance to heat transfer. This phenomenon of product buildup on the heat transfer surface is called fouling. A similar phenomenon is observed when a liquid is brought into contact with subcooled surfaces. Fouling of heat transfer surfaces not only increases thermal resistance but may also restrict fluid flow. Furthermore, valuable components of the food are lost to the fouled layer. Fouling is remedied by cleaning heating surfaces with strong chemicals that are also environmental pollutants.

Fouling is of major concern in the chemical process industries. Its role is even more pronounced in the food industry where many heat-sensitive components of foods can easily deposit on a heat-transfer surface. As a result, factory operations involving heating or cooling require frequent cleaning, often on a daily basis. Some of the common types of fouling and their underlying mechanisms are shown in Table 4.3.

The fouling layer often has a composition different from the liquid stream that causes fouling. With milk, which has a protein content of around 3%, the fouling deposits resulting at temperatures less than 110°C contain 50 to 60% protein and 30 to 35% minerals. About half of the protein in the fouled layer is β-lactoglobulin. When the temperature of the milk increases above 70 to 74°C, protein denaturation increases. The protein (β-lactoglobulin) first unfolds and the reactive sulphydryl groups are exposed. This is followed by polymerization (or aggregation) of the molecule with itself or other proteins including α-lactoglobulin.

Table 4.3 Common Mechanisms in Fouling of Heat Exchange Surfaces

Type of fouling	Fouling mechanism
Precipitation	Precipitation of dissolved substances. Salts such as $CaSO_4$, $CaCO_3$ cause scaling.
Chemical reaction	Surface material acts as a reactant; chemical reactions of proteins, sugars, and fats.
Particulate	Accumulation of fine particles suspended in the processed fluids on the heat transfer surface.
Biological	Attachment of organisms both macro and micro on heat transfer surface.
Freezing	Solidification of liquid components on subcooled surfaces.
Corrosion	Heat transfer surface reacts with ambient and corrodes.

Fouling results from a complex series of reactions, and in heating processes these reactions are accelerated with temperature. To compensate for the reduced rate of heat transfer due to the increased thermal resistance, a larger heat-transfer surface area is required, which increases the cost of heat exchange equipment. For an operating heat exchanger with a fouled surface, the reduced rate of heat transfer is compensated for by using higher temperature gradients across the heat transfer medium. Consequently, the energy requirements to operate heat exchange equipment increase significantly. It has been estimated that the annual worldwide cost of fouling to process industries is several billion dollars.

Let us examine the role of fouling on heat transfer by considering the rates of heat transfer in a clean pipe and in one that has been fouled on both the inside and outside surfaces (Fig. 4.27). Assuming that the deposited layers are thin, the convective heat transfer coefficient on the inside surface of the fouled pipe, h_{fi}, will be same as that on the inside surface of the clean pipe h_{ci}. The same will hold true for the convective heat transfer coefficients on the outside surfaces; that is, $h_{fo} = h_{co}$. Similarly, the inside surface area of the fouled pipe, $A_{fi} = A_{ci} = A_i$, and, for the outside surface area, $A_{fo} = A_{co} = A_o$.

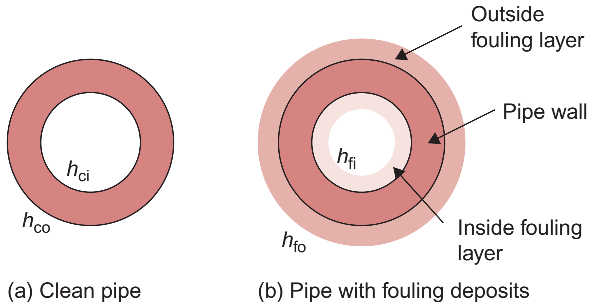


Figure 4.27 A pipe with fouling deposits on the inside and outside surfaces

Using Equation (4.85), we write an equation for the overall heat transfer coefficient based on the outside area for the clean pipe,

$$\frac{1}{U_{co}A_o} = \frac{1}{h_i A_i} + \frac{\ln\left(\frac{r_o}{r_i}\right)}{2\pi Lk} + \frac{1}{h_o A_o} \tag{4.87}$$

or,

$$\frac{1}{U_{co}} = \frac{A_o}{h_i A_i} + \frac{A_o \ln\left(\frac{r_o}{r_i}\right)}{2\pi Lk} + \frac{1}{h_o} \tag{4.88}$$

Next, we will consider a pipe that is fouled on both the inside and outside surface. As shown in Figure 4.27b, the fouling resistance due to the deposited layer on the inside of the pipe is R_{fi} (m^2 °C/W) and for the outside pipe it is R_{fo} (m^2 °C/W). Then for the fouled pipes,

$$\frac{1}{U_{fo}A_o} = \frac{1}{h_i A_i} + \frac{R_{fi}}{A_i} + \frac{\ln\left(\frac{r_o}{r_i}\right)}{2\pi Lk} + \frac{R_{fo}}{A_o} + \frac{1}{h_o A_o} \tag{4.89}$$

or,

$$\frac{1}{U_{fo}} = \frac{A_o}{h_i A_i} + \frac{R_{fi}A_o}{A_i} + \frac{A_o \ln\left(\frac{r_o}{r_i}\right)}{2\pi Lk} + R_{fo} + \frac{1}{h_o} \tag{4.90}$$

Since R_{fi} and R_{fo} are difficult to determine separately, the two terms for fouling resistance in Equations (4.88) are combined to get the total resistance due to fouling, R_{ft},

$$R_{ft} = \frac{A_o}{A_i} R_{fi} + R_{fo} \tag{4.91}$$

Then

$$\frac{1}{U_{\text{fo}}} = \frac{A_{\text{o}}}{h_{\text{i}}A_{\text{i}}} + \frac{A_{\text{o}} \ln\left(\frac{r_{\text{o}}}{r_{\text{i}}}\right)}{2\pi Lk} + R_{\text{ft}} + \frac{1}{h_{\text{o}}} \tag{4.92}$$

Combining Equations (4.88) and (4.92), we get

$$\frac{1}{U_{\text{fo}}} = \frac{1}{U_{\text{co}}} + R_{\text{ft}} \tag{4.93}$$

In industrial practice, a term called Cleaning factor (C_{F}) is defined as a ratio between the two overall heat transfer coefficients, or,

$$C_{\text{F}} = \frac{U_{\text{fo}}}{U_{\text{co}}} \tag{4.94}$$

Note that the value of C_{F} is less than 1. Substituting Equation (4.93) in Equation (4.94), and rearranging terms we can write the fouling resistance in terms of the cleaning factor.

$$R_{\text{ft}} = \frac{1}{U_{\text{co}}}\left(\frac{1}{C_{\text{F}}} - 1\right) \tag{4.95}$$

In designing heat exchangers, we may want to determine the extra area required due to expected fouling of the heat transfer area. For this purpose, we consider the rates of heat transfer in the clean and fouled surface to be the same. Thus,

$$q = U_{\text{co}}A_{\text{co}}\Delta T_{\text{m}} = U_{\text{fo}}A_{\text{fo}}\Delta T_{\text{m}} \tag{4.96}$$

Then eliminating ΔT_{m} in the preceding equation,

$$U_{\text{co}}A_{\text{co}} = U_{\text{fo}}A_{\text{fo}} \tag{4.97}$$

or,

$$\frac{U_{\text{co}}}{U_{\text{fo}}} = \frac{A_{\text{fo}}}{A_{\text{co}}} \tag{4.98}$$

Combining Equations (4.94), (4.95), and (4.98), and rearranging terms, we get

$$R_{\text{ft}} = \frac{1}{U_{\text{co}}}\left(\frac{A_{\text{fo}}}{A_{\text{co}}} - 1\right) \tag{4.99}$$

In Equation (4.99) the term $((A_{fo}/A_{co}) - 1)$ multiplied by 100 is the percent extra area required for the fouled surface compared with the clean surface. In the following example, we will use the previous derived relations to develop a graph of percent increase in area required to compensate for fouling for different values of overall heat transfer coefficients.

Example 4.17

Using a spreadsheet, develop a chart that shows the relationship between the total fouling resistance as a function of overall heat transfer coefficients of 1000 to 5000 W/m^{2}K for cleanliness factors of 0.8, 0.85, 0.9, 0.95. Also, develop a chart that demonstrates the increase in surface area required for fouling resistances of 0.0001, 0.001, 0.01, and 0.05 m^{2}K/W for an overall heat transfer coefficient varying from 1 to 10,000 W/m^{2}K.

Given

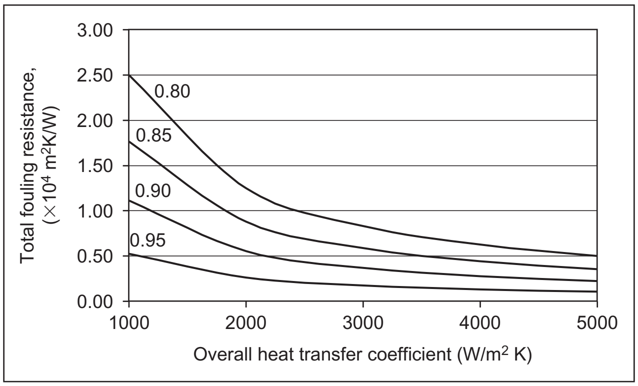
Part (a)

Overall heat transfer coefficient = 1000, 2000, 3000, 4000 and 5000 W/m^{2}K

Cleanliness factor = 0.8, 0.85, 0.9, 0.95

	A	B	C	D	E
1			Cleanliness Factor		
2	Overall Heat Transfer Coefficient	0.80	0.85	0.90	0.95
3	1000	2.50	1.76	1.11	0.53
4	2000	1.25	0.88	0.56	0.26
5	3000	0.83	0.59	0.37	0.18
6	4000	0.63	0.44	0.28	0.13
7	5000	0.50	0.35	0.22	0.11

1) Enter values as shown in cells A3:A7 and B2:E2
2) Enter=(1/$A3)*(1/B$2-1)*10000 in cell B3 and copy in cells B4:B7 and C3:E7



■ **Figure E4.14** Total fouling resistance as a function of overall heat transfer coefficient for different cleanliness factors.

Part (b)

Fouling resistance = 0.0001, 0.001, 0.01 and 0.05 m^2K/W

Overall heat transfer coefficient = 1, 10, 100, 1000, 10,000 W/m^2K

Approach

*We will develop two spreadsheets using Excel™. First, we will calculate the fouling resistance for **Part (a)** and the required increase in surface area for **Part (b)**. Then we will create a plot from the calculated data.*

Solution

The first spreadsheet is developed using Equation (4.95) for the fouling resistance R_f as a function of the overall heat transfer coefficient for a clean pipe. The second spreadsheet is developed using Equation (4.99), and we will plot $\left(\frac{A_{fo}}{A_{co}} - 1\right) \times 100$ against overall heat transfer coefficient on a log-log scale. As we observe, from Figure E4.14, as the cleanliness factor decreases, the fouling resistance increases for the same value of clean-pipe overall heat transfer coefficient. This effect is more pronounced at low values of overall heat transfer coefficient. Similarly, from Figure E4.15, for a small increase in the

	A	B	C	D	E
1			Fouling resistance (m^2K/W)		
2	Overall Heat Transfer Coefficient	0.0001	0.001	0.01	0.05
3	1	0.01	0.1	1	5
4	10	0.1	1	10	50
5	100	1	10	100	500
6	1000	10	100	1000	5000
7	10000	100	1000	10000	50000

1) Enter values as shown in cells A3:A7 and B2:E2
2) Enter = $A3*B$2*100 in cell B3 and
paste in cells B4:B7 and C3:E7

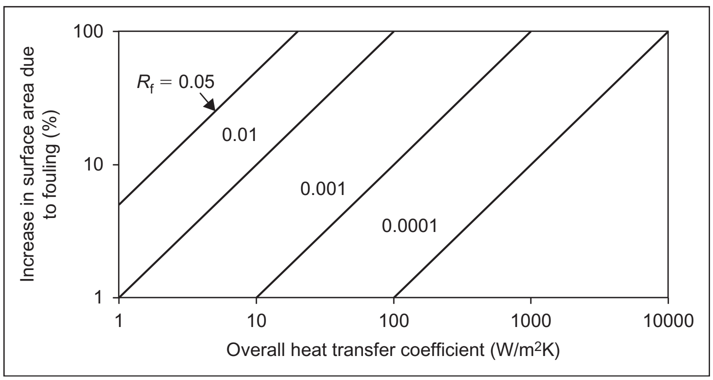


Figure E4.15 Increase in surface area due to fouling.

resistance factor, for the same overall heat transfer coefficient, a much larger surface area is required. These graphs show that the resistance due to fouling has a substantial effect on heat transfer, and larger surface areas are required with increasing fouling.

4.4.7 Design of a Tubular Heat Exchanger

In Section 4.1, we examined a variety of heat exchange equipment used in the food process industry. Recall that a number of different geometrical configurations are used in the design of heat exchange equipment, such as tubular, plate, and scraped surface heat exchangers. The primary objective in using a heat exchanger is to transfer thermal energy from one fluid to another. In this section we will develop calculations necessary to design a tubular heat exchanger.

One of the key objectives in calculations involving a heat exchanger is to determine the required heat transfer area for a given application. We will use the following assumptions:

1. Heat transfer is under steady-state conditions.
2. The overall heat-transfer coefficient is constant throughout the length of pipe.
3. There is no axial conduction of heat in the metal pipe.
4. The heat exchanger is well insulated. The heat exchange is between the two liquid streams flowing in the heat exchanger. There is negligible heat loss to the surroundings.

Recall from Chapter 1 that change in heat energy in a fluid stream, if its temperature changes from T_1 to T_2, is expressed as:

$$q = \dot{m}c_p(T_1 - T_2) \tag{4.100}$$

where $\dot{m}$ is mass flow rate of a fluid (kg/s), c_p is specific heat of a fluid (kJ/[kg °C]), and the temperature change of a fluid is from some inlet temperature T_1 to an exit temperature T_2.

Consider a tubular heat exchanger, as shown in Figure 4.28. A hot fluid, H, enters the heat exchanger at location (1) and it flows through the inner pipe, exiting at location (2). Its temperature decreases from $T_{H,inlet}$ to $T_{H,exit}$. The second fluid, C, is a cold fluid that enters the annular space between the outer and inner pipes of the tubular heat exchanger at location (1) and exits at location (2). Its temperature increases from $T_{C,inlet}$ to $T_{C,exit}$. The outer pipe of the heat exchanger is

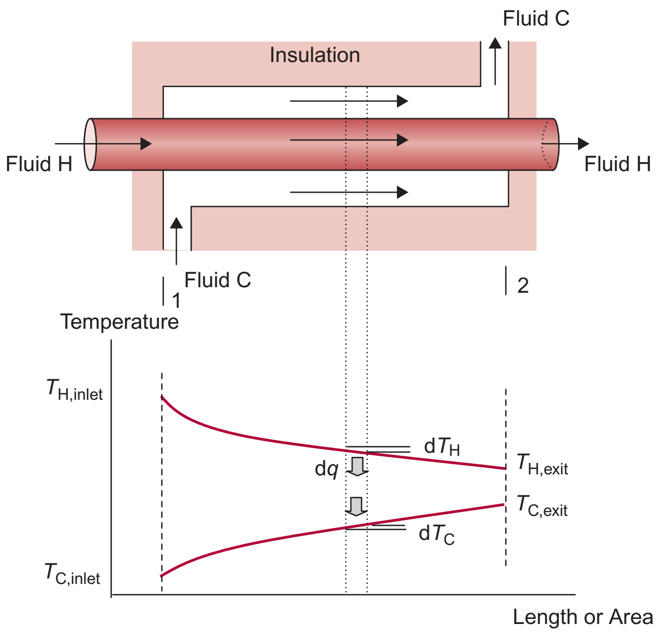


Figure 4.28 A concurrent flow heat exchanger and temperature plots.

covered with an insulation to prevent any heat exchange with the surroundings. Because the heat transfer occurs only between fluids *H* and *C*, the decrease in the heat energy of fluid *H* must equal the increase in the energy of fluid *C*. Therefore, conducting an energy balance, the rate of heat transfer between the fluids is:

$$q = \dot{m}_H c_{pH}(T_{H,inlet} - T_{H,exit}) = \dot{m}_C c_{pC}(T_{C,exit} - T_{C,inlet}) \qquad (4.101)$$

where c_{pH} is the specific heat of the hot fluid (kJ/[kg °C]), c_{pC} is the specific heat of the cold fluid (kJ/[kg °C]), $\dot{m}_H$ is the mass flow rate of the hot fluid (kg/s), and $\dot{m}_C$ is the mass flow rate of the cold fluid (kg/s).

Equation (4.101) is useful if we are interested in determining the inlet and exit temperatures of the two fluid streams. Furthermore, we may use this equation to determine the mass flow rate of either fluid stream, provided all other conditions are known. But, this equation does not provide us with any information about the size of the heat exchanger required for accomplishing a desired rate of heat transfer, and we cannot use it to determine how much thermal resistance to heat transfer exists between the two fluid streams. For those questions, we need to determine heat transfer perpendicular to the flow of the fluid streams, as discussed in the following.

Consider a thin slice of the heat exchanger, as shown in Figure 4.28. We want to determine the rate of heat transfer from fluid H to C, perpendicular to the direction of the fluid streams. For this thin slice of the heat exchanger, the rate of heat transfer, $\mathrm{d}q$, from fluid H to fluid C may be expressed as:

$$\mathrm{d}q = U\,\Delta T\,\mathrm{d}A \tag{4.102}$$

where ΔT is the temperature difference between fluid H and fluid C. Note that this temperature difference, ΔT, varies from location (1) to (2) of the heat exchanger. At the inlet of the fluid streams, location (1), the temperature difference, ΔT, is $T_{\mathrm{H,inlet}} - T_{\mathrm{C,inlet}}$ and on the exit side, location (2), it is $T_{\mathrm{H,exit}} - T_{\mathrm{C,exit}}$ (Fig. 4.28). To solve Equation (4.102) we can substitute only one value of ΔT, or its average value that represents the temperature gradient perpendicular to the direction of the flow. Although it may be tempting to take an arithmetic average of the two ΔT values from locations (1) and (2), the arithmetic average value will be incorrect because, as seen in Figure 4.28, the temperature plots are nonlinear. Therefore, we will develop the following mathematical analysis to determine a value of ΔT that will correctly identify the "average" temperature difference between the fluids H and C as they flow through the heat exchanger.

The temperature difference, ΔT, between the two fluids H and C is

$$\Delta T = T_{\mathrm{H}} - T_{\mathrm{C}} \tag{4.103}$$

where T_{H} is the temperature of the hot stream and T_{C} is that of the cold stream. For a small differential ring element as shown in Figure 4.28, using energy balance for the hot stream H we get

$$\mathrm{d}q = -\,\dot{m}_{\mathrm{H}} c_{\mathrm{pH}} \mathrm{d}T_{\mathrm{H}} \tag{4.104}$$

and, for cold stream C in the differential element,

$$\mathrm{d}q = \dot{m}_{\mathrm{C}} c_{\mathrm{pC}} \mathrm{d}T_{\mathrm{C}} \tag{4.105}$$

In Equation (4.104), $\mathrm{d}T_{\mathrm{H}}$ is a negative quantity; therefore, we added a negative sign to obtain a positive value for $\mathrm{d}q$. Solving for $\mathrm{d}T_{\mathrm{H}}$ and $\mathrm{d}T_{\mathrm{C}}$, we obtain

$$\mathrm{d}T_{\mathrm{H}} = -\frac{\mathrm{d}q}{\dot{m}_{\mathrm{H}} c_{\mathrm{pH}}} \tag{4.106}$$

and

$$dT_C = -\frac{dq}{\dot{m}_C c_{pC}} \tag{4.107}$$

Then, subtracting Equation (4.107) from Equation (4.106),

$$dT_H - dT_C = d(T_H - T_C) = -dq\left(\frac{1}{\dot{m}_H c_{pH}} + \frac{1}{\dot{m}_C c_{pC}}\right) \tag{4.108}$$

Using Equations (4.102) and (4.103), and substituting in Equation (4.108),

$$\frac{d(T_H - T_C)}{(T_H - T_C)} = -U\left(\frac{1}{\dot{m}_H c_{pH}} + \frac{1}{\dot{m}_C c_{pC}}\right)dA \tag{4.109}$$

Integrating Equation (4.109) from locations (1) to (2) shown in Figure 4.28,

$$\ln\frac{(T_{H,exit} - T_{C,exit})}{(T_{H,inlet} - T_{C,inlet})} = -UA\left(\frac{1}{\dot{m}_H c_{pH}} + \frac{1}{\dot{m}_C c_{pC}}\right) \tag{4.110}$$

Noting that

$$\begin{aligned} T_{H,inlet} - T_{C,inlet} &= \Delta T_1 \\ T_{H,exit} - T_{C,exit} &= \Delta T_2 \end{aligned} \tag{4.111}$$

we get

$$\ln\frac{\Delta T_2}{\Delta T_1} = -UA\left(\frac{1}{\dot{m}_H c_{pH}} + \frac{1}{\dot{m}_C c_{pC}}\right) \tag{4.112}$$

Substituting Equation (4.101) in Equation (4.112),

$$\ln\left(\frac{\Delta T_2}{\Delta T_1}\right) = -UA\left(\frac{T_{H,inlet} - T_{H,exit}}{q} + \frac{T_{C,exit} - T_{C,inlet}}{q}\right) \tag{4.113}$$

Rearranging terms in Equation (4.113),

$$\ln\left(\frac{\Delta T_2}{\Delta T_1}\right) = \frac{-UA}{q}\left[(T_{H,inlet} - T_{C,inlet}) - (T_{H,exit} - T_{C,exit})\right] \tag{4.114}$$

Substituting Equation (4.111) in Equation (4.114), we obtain

$$\ln\left(\frac{\Delta T_2}{\Delta T_1}\right) = -\frac{UA}{q}(\Delta T_1 - \Delta T_2) \tag{4.115}$$

Rearranging terms,

$$q = UA\frac{\Delta T_2 - \Delta T_1}{\ln\left(\dfrac{\Delta T_2}{\Delta T_1}\right)} \tag{4.116}$$

$$q = UA(\Delta T_{\mathrm{lm}}) \tag{4.117}$$

where

$$\Delta T_{\mathrm{lm}} = \frac{\Delta T_2 - \Delta T_2}{\ln\left(\dfrac{\Delta T_2}{\Delta T_1}\right)} \tag{4.118}$$

ΔT_{lm} is called the log mean temperature difference (LMTD). Equation (4.117) is used to design a heat exchanger and determine its area and the overall resistance to heat transfer, as illustrated in Examples 4.18 and 4.19.

Example 4.18

A liquid food (specific heat = 4.0 kJ/[kg °C]) flows in the inner pipe of a double-pipe heat exchanger. The liquid food enters the heat exchanger at 20°C and exits at 60°C (Fig. E4.16). The flow rate of the liquid food is 0.5 kg/s. In the annular section, hot water at 90°C enters the heat exchanger and flows countercurrently at a flow rate of 1 kg/s. The average specific heat of water is 4.18 kJ/(kg °C). Assume steady-state conditions.

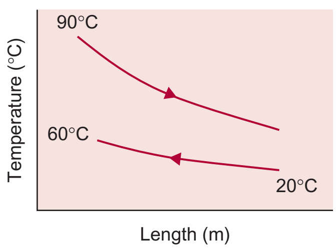


Figure E4.16 A countercurrent flow heat exchanger with unknown exit temperatures.

1. Calculate the exit temperature of water.
2. Calculate log-mean temperature difference.
3. If the average overall heat transfer coefficient is 2000 W/(m^2 °C) and the diameter of the inner pipe is 5 cm, calculate the length of the heat exchanger.
4. Repeat these calculations for parallel-flow configuration.

Given

Liquid food:

Exit temperature = 60°C

Inlet temperature = 20°C

Specific heat = 4.0 kJ/(kg °C)
Flow rate = 0.5 kg/s

Water:

Inlet temperature = 90°C
Specific heat = 4.18 kJ/(kg °C)
Flow rate = 1.0 kg/s

Heat exchanger:

Diameter of inner pipe = 5 cm
Flow = countercurrent

Approach

We will first calculate the exit temperature of hot water by using a simple heat balance equation. Then we will compute log-mean temperature difference. The length of the heat exchanger will be determined from Equation (4.117). The solution will be repeated for parallel-flow configuration to obtain a new value for log-mean temperature difference and length of the heat exchanger.

Solution

1. Using a simple heat balance,

$$q = \dot{m}_C c_{pC} \Delta T_C = \dot{m}_h c_{ph} \Delta T_h$$
$$= (0.5\ \text{kg/s})(4\text{kJ}/[\text{kg °C}])(60°\text{C} - 20°\text{C})$$
$$= (1\ \text{kg/s})(4.18\ \text{kJ}/[\text{kg °C}])(90°\text{C} - T_e°\text{C})$$
$$T_e = 70.9°\text{C}$$

2. The exit temperature of water is 70.9°C.
3. From Equation (4.118)

$$(\Delta T)_{lm} = \frac{\Delta(T)_1 - \Delta(T)_2}{\ln\left[\dfrac{\Delta(T)_1}{\Delta(T)_2}\right]} = \frac{(70.9 - 20) - (90 - 60)}{\ln\left(\dfrac{50.9}{30}\right)}$$
$$= 39.5°\text{C}$$

4. The log-mean temperature difference is 39.5°C.
5. From Equation (4.117),

$$q = UA(\Delta T)_{lm} = U\,\pi\, D_i L(\Delta T)_{lm}$$

where q, from step (1), is

$$q = (0.5\ \text{kg/s})(4\ \text{kJ}/[\text{kg °C}])(60°\text{C} - 20°\text{C}) = 80\ \text{kJ/s}$$

Thus,

$$L = \frac{(80\ kJ/s)(1000\ J/kJ)}{(\pi)(0.05\ m)(39.5°C)(2000\ W/[m^2\ °C])} = 6.45\ m$$

6. *The length of the heat exchanger, when operated counter-currently, is 6.5 m.*
7. *For parallel-flow operation, the system diagram will be as shown in Figure E4.16 .*
8. *Assuming that for parallel flow the exit temperature will be the same as for counterflow, $T_e = 70.9°C$.*
9. *Log-mean temperature difference is calculated from Equation (4.118).*

$$(\Delta T)_{lm} = \frac{(90 - 20) - (70.9 - 60)}{\ln\left(\frac{90 - 20}{70.9 - 60}\right)} = 31.8°C$$

10. *The log-mean temperature difference for parallel flow is 31.8°C, about 8°C less than that for the countercurrent flow arrangement.*
11. *The length can be computed as in step (5).*

$$L = \frac{(80\ kJ/s)(1000\ J/kJ)}{(\pi)(0.05\ m)(31.8°C)(2000\ W/[m^2\ °C])} = 8\ m$$

12. *The length of the heat exchanger, when operated as parallel flow, is 8 m. This length of heat exchanger is longer by 1.55 m to obtain the same exit temperature of hot-water stream as for the counterflow arrangement.*

Example 4.19

Steam with 90% quality, at a pressure of 143.27 kPa, is condensing in the outer annular space of a 5 m-long double-pipe heat exchanger (Fig. E.17 and E4.18). Liquid food is flowing at a rate of 0.5 kg/s in the inner pipe. The inner pipe has an inside diameter of 5 cm. The specific heat of the liquid food is 3.9 kJ/(kg °C). The inlet temperature of the liquid food is 40°C and the exit temperature is 80°C.

a. Calculate the average overall heat-transfer coefficient.
b. If the resistance to conductive heat transfer caused by the inner steel pipe is negligible, and the convective heat-transfer coefficient on the steam side is very large (approaches infinity), estimate the convective heat-transfer coefficient for the liquid food in the inside pipe.

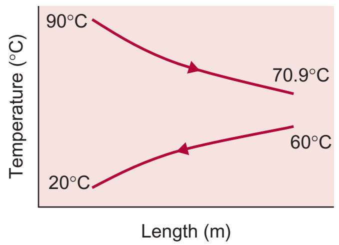


Figure E4.17 Temperature plots for a counterflow heat exchanger.

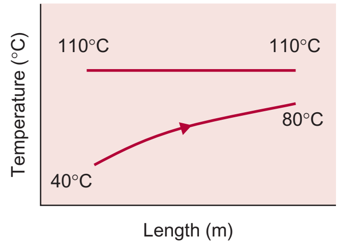


Figure E4.18 Temperature plots for a parallel-flow heat exchanger.

Given

Steam pressure = 143.27 kPa
Length = 5 m
Flow rate of liquid = 0.5 kg/s
Inside diameter = 0.05 m
Specific heat = 3.9 kJ/(kg °C)
Product inlet temperature = 40°C
Product exit temperature = 80°C

Approach

We will obtain steam temperature from Table A.4.2. We also note that steam quality has no effect on the steam condensation temperature. We will calculate the heat required to raise the liquid food temperature from 40 to 80°C. Next, we will calculate log-mean temperature difference. Then we will obtain the overall heat-transfer coefficient by equating the heat gain of the liquid food and heat transfer across the pipe wall, from steam to the liquid food.

Solution

Part (a)

1. From steam table (Table A.4.2), steam temperature = 110°C.
2.

$$q = \dot{m}c_p\Delta T$$
$$= (0.5\ \text{kg/s})(3.9\ \text{kJ/[kg °C]})(1000\ \text{J/kJ})(80°\text{C} - 40°\text{C})$$
$$= 78{,}000\ \text{J/s}$$

3.

$$q = UA(\Delta T)_{lm} = \dot{m}c_p\Delta T$$
$$(\Delta T)_{lm} = \frac{(110-40)-(110-80)}{\ln\left(\dfrac{110-40}{110-80}\right)} = 47.2°\text{C}$$

and

$$A = \pi(0.05)(5) = 0.785\ \text{m}^2$$

4.

$$U = \frac{\dot{m}c_p\Delta T}{A(\Delta T)_{lm}} = \frac{(78{,}800\ \text{J/s})}{(0.785\ \text{m}^2)(47.2°\text{C})} = 2105\ \text{W/(m}^2\ °\text{C)}$$

5. Overall heat-transfer coefficient = 2105 W/(m^2 °C)

Part (b)

The overall heat-transfer equation may be written as follows:

$$\frac{1}{U_i A_i} = \frac{1}{h_i A_i} + \frac{\ln\left(\frac{r_o}{r_i}\right)}{2\pi kL} + \frac{1}{h_o A_o}$$

The second term on the right-hand side of the above equation is zero, since the resistance offered by steel to conductive heat transfer is considered negligible. Likewise, the third term is zero, because the convective heat-transfer coefficient on the steam side is very large.

Therefore,

$$U_i = h_i$$

or

$$h_i = 2105\ W/(m^2\ °C)$$

4.4.8 The Effectiveness-*NTU* Method for Designing Heat Exchangers

In the preceding section, we used the log-mean-temperature-difference (LMTD) approach to design a heat exchanger. The LMTD approach works well when we are designing a new heat exchanger, where the temperatures of the fluid streams at the inlet and exit are known and we are interested in determining the size of the heat exchanger (in terms of the heat transfer area, length, and diameter of the pipe). However, in other situations when the size of the heat exchanger, and the inlet temperatures of the product and heating/cooling streams are known but the exit temperatures of the two streams are unknown, the LMTD approach can be used but the solution requires iterative procedures and becomes tedious. For this purpose, another calculation technique called the effectiveness-*NTU* method is easier to use. This method involves three dimensionless quantities, namely the heat capacity rate ratio, heat exchanger effectiveness, and number of transfer units (*NTU*).

4.4.8.1 *Heat Capacity Rate Ratio, C**

A heat capacity rate of a liquid stream is obtained as a product of the mass flow rate and specific heat capacity. Thus, for the hot and cold streams the heat capacity rates are, respectively:

$$C_H = \dot{m}_H c_{PH} \tag{4.119}$$

$$C_C = \dot{m}_C c_{PC} \tag{4.120}$$

These two quantities are evaluated using the given data in a problem; the smaller of the two quantities is called C_{min} and the larger C_{max}.

The heat capacity rate ratio, C^*, is defined as

$$C^* = \frac{C_{min}}{C_{max}} \tag{4.121}$$

4.4.8.2 Heat Exchanger Effectiveness, ε_E

The heat exchanger effectiveness is a ratio of the actual rate of heat transfer accomplished and the maximum attainable rate of heat transfer for a given heat exchanger. The heat exchanger effectiveness, ε_E, is defined as

$$\varepsilon_E = \frac{q_{actual}}{q_{max}} \tag{4.122}$$

The actual rate of heat transfer can be determined for both hot and cold streams as

$$q_{actual} = C_H(T_{H,inlet} - T_{H,exit}) = C_C(T_{C,exit} - T_{C,inlet}) \tag{4.123}$$

And, the maximum rate of attainable heat transfer is obtained by observing that in any heat exchanger the maximum possible temperature difference is between the temperatures of hot and cold streams at the inlet. This temperature difference is multiplied with the minimum heat capacity rate, C_{min}, to obtain q_{max}:

$$q_{max} = C_{min}(T_{H,inlet} - T_{C,inlet}) \tag{4.124}$$

Thus, from Equation (4.122),

$$q_{actual} = \varepsilon_E q_{max} = \varepsilon_E C_{min}(T_{H,inlet} - T_{C,inlet}) \tag{4.125}$$

Table 4.4 Effectiveness-NTU Relations for Heat Exchangers

Type of heat exchanger	Effectiveness relation
Double pipe Concurrent flow	$\varepsilon_E = \dfrac{1 - \exp[-NTU(1 + C^*)]}{1 + C}$
Double pipe Countercurrent flow	$\varepsilon_E = \dfrac{1 - \exp[-NTU(1 + C^*)]}{1 - C^* \exp[-NTU(1 - C^*)]}$
Shell and tube: One-shell pass 2, 4, 6... tube passes	$\varepsilon_E = \dfrac{2}{1 + C^* + \sqrt{1 + C^{*2}} \dfrac{1 + \exp\left[-NTU\sqrt{1 + C^{*2}}\right]}{1 - \exp\left[-NTU\sqrt{1 + C^{*2}}\right]}}$
Plate heat exchanger	$\varepsilon_E = \dfrac{\exp[(1 - C^*) \times NTU] - 1}{\exp[(1 - C^*) \times NTU] - C^*}$
All heat exchangers, $C^* = 0$	$\varepsilon_E = 1 - \exp(-NTU)$

4.4.8.3 *Number of Transfer Units, NTU*

The number of transfer units provides a measure of the heat transfer surface area for a given overall heat transfer coefficient and minimum heat capacity rate. It is expressed as

$$NTU = \frac{UA}{C_{min}} \tag{4.126}$$

where A is the heat transfer area (m^2), U is the overall heat transfer coefficient (W/m^2 °C) based on the selected area (see Section 4.4.5), and C_{min} is the minimum heat capacity rate (W/°C).

Relationships between NTU and effectiveness can be obtained for different types of heat exchangers with prescribed flow conditions (such as counterflow or concurrent flow). These relationships include the heat capacity rate ratios. Some of these relationships for commonly used heat exchangers are shown in Tables 4.4 and 4.5. In Table 4.4, the exchanger effectiveness is given in terms of NTU, and in Table 4.5, the NTU values are given as a function of the exchanger effectiveness. The steps involved in using the effectiveness-NTU method are illustrated in the following example.

Table 4.5 NTU-Effectiveness Relations for Heat Exchangers

Type of heat exchanger	NTU relation
Double pipe Concurrent flow	$NTU = -\dfrac{\ln[1-\varepsilon_E(1+C^*)]}{1+C^*}$
Double pipe Countercurrent flow	$NTU = \dfrac{1}{1-C^*}\ln\left[\dfrac{1-C^*\varepsilon_E}{1-\varepsilon_E}\right] \quad (C^*<1)$ $NTU = \dfrac{\varepsilon_E}{1-\varepsilon_E} \quad (C^*=1)$
Shell and tube: One-shell pass 2, 4, 6... tube passes	$NTU = \dfrac{1}{\sqrt{1+C^{*2}}}\ln\dfrac{2-\varepsilon_E\left[1+C^*-\sqrt{1+C^{*2}}\right]}{2-\varepsilon_E\left[1+C^*+\sqrt{1+C^{*2}}\right]}$
Plate heat exchanger	$NTU = \dfrac{\ln\left[\dfrac{(1-C^*)}{(1-\varepsilon_E)}\right]}{(1-C^*)}$
All heat exchangers, $C^*=0$	$NTU = -\ln(1-\varepsilon_E)$

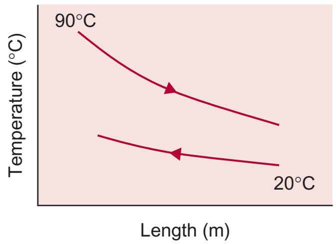
Example 4.20

We will use some of the data from Example 4.18 to show the use of the effectiveness-NTU method in solving problems when both exit temperatures are unknown. A liquid food (specific heat = 4.0 kJ/[kg °C]) flows in the inner pipe of a double-pipe heat exchanger. The liquid food enters the heat exchanger at 20°C. The flow rate of the liquid food is 0.5 kg/s. In the annular section, hot water at 90°C enters the heat exchanger and flows in countercurrent direction at a flow rate of 1 kg/s. The average specific heat of water is 4.18 kJ/(kg °C). The average overall heat transfer coefficient based on the inside area is 2000 W/(m^2 °C), and the diameter of the inner pipe is 5 cm and length is 6.45 m. Assume steady state conditions. Calculate the exit temperature of liquid food and water.

Given

Liquid food:

Inlet temperature = 20°C
Specific heat = 4.0 kJ/(kg °C)
Flow rate = 0.5 kg/s



■ **Figure E4.19** Temperature plots for a double-pipe heat exchanger.

Water:

Inlet temperature = 90°C

Specific heat = 4.18 kJ/(kg °C)

Flow rate = 1.0 kg/s

Heat exchanger:

Diameter of inner pipe = 5 cm

Length of inner pipe = 6.45 m

Overall heat transfer coefficient = 2000 W/m^2 °C

Flow = countercurrent

Approach

Since exit temperatures of both streams are unknown (Fig. E4.19), we will use the effectiveness-NTU method. We will first calculate the maximum and minimum heat capacity rates. The two heat capacity rates will be used to calculate the heat capacity rate ratio, C. Next, we will obtain NTU from the given overall heat transfer coefficient, heat exchanger area, and the calculated value of C_{min}. Using the calculated value of NTU, we will use an appropriate table to determine the exchanger effectiveness. The definition of the exchanger effectiveness will be used to determine q_{actual}, and the unknown temperatures, $T_{H,exit}$ and $T_{L,exit}$.*

Solution

1. *The heat capacity rates for the hot water and liquid food are, respectively,*

$$\begin{aligned} C_H &= \dot{m}_H c_{pH} \\ &= (1.0\ \text{kg/s})(4.18\ \text{kJ/[kg °C]}) \\ &= 4.18\ \text{kW/°C} \\ C_L &= \dot{m}_L c_{pL} \\ &= (0.5\ \text{kg/s})(4\ \text{kJ/[kg °C]}) \\ &= 2\ \text{kW/°C} \end{aligned}$$

Therefore, using the smaller value of C between the above calculated values C_H and C_L,

$$C_{\min} = 2\ \text{kW/°C}$$

Then, C is obtained as*

$$C^* = \frac{2}{4.18} = 0.4785$$

2. *The NTU value is obtained from Equation (4.126) as*

$$NTU = \frac{UA}{C_{min}} = \frac{(2000\ W/m^2\ °C)(\pi)(0.05\ m)(6.45m)}{(2\ kW/°C)(1000\ W/kW)}$$

$$NTU = 1.0132$$

3. *From Table 4.4 we select an expression for ε_E for tubular heat exchanger, for counterflow, and substitute known terms as*

$$\varepsilon_E = \frac{1 - e^{(-1.0132(1-0.4785))}}{1 - 0.4785e^{(-1.0132(1-04785))}}$$

$$\varepsilon_E = 0.5717$$

According to Equation (4.124)

$$q_{max} = (2kW/°C)(90 - 20)(°C) = 140\ kW$$

Since $\varepsilon_E = \dfrac{q_{actual}}{q_{max}}$

$$q_{actual} = 0.5717 \times 140(kW) = 80.038\ kW$$

For hot water stream:

$$q_{actual} = 4.18(kW/°C) \times (90 - T_{H,exit})(°C) = 80.038\ kW$$

$$T_{H,exit} = 90 - 19.15 = 70.85°C$$

Similarly, for liquid food stream:

$$q_{actual} = 2(kW/°C) \times (T_{L,exit} - 20)(°C) = 80.038\ kW$$

$$T_{L,exit} = 40.019 + 20 = 60°C$$

The calculated exit temperatures of hot water and product streams are 70.85°C and 60°C, respectively. These values are comparable to those given in Example 4.18.

4.4.9 Design of a Plate Heat Exchanger

As discussed in Section 4.1.1, plate heat exchangers are commonly used in the food industry. The design of a plate heat exchanger requires certain relationships that are often unique to the plates used in the heat exchanger. Some of the required information for design purposes is closely guarded by the equipment manufacturers and is not readily available. We will consider a general design approach with some of the key relationships required in designing plate heat exchangers. First, it is important to understand the type of flow pattern present inside a plate heat exchanger.

Figure 4.2 shows an arrangement of plates with two liquid streams, product and heating/cooling medium, entering and exiting through their respective ports. The assembly of a plate heat exchanger involves placing the required number of plates in a frame and tightening the end bolts so that a fixed gap space is created between the plates. The gaps between the plates create channels through which the fluid streams flow. The fluid streams enter and exit through the port holes (Fig. 4.2b).

Each plate contains a gasket that helps orient the direction of the flow stream through the channel. The assembly of plates with gaskets is done in such a manner that it allows the product stream to move in one channel while the heating/cooling stream moves in the neighboring channel. Thus, the flow of product and heating/cooling streams alternates through the channels and they never come into direct contact (Fig. 4.2).

The heat exchange between the two liquid streams in a plate heat exchanger is across the plates, normal to the direction of the flow. Therefore, the plates are made as thin as possible to minimize resistance to heat transfer while maintaining the physical integrity of the plates. The plates are corrugated with a pattern that promotes turbulence in the liquid stream. Several types of patterns are used for the corrugations stamped on the plates, the most common one being a herringbone type design called the Chevron design (Fig. 4.3). The plates used in food processing applications are made of stainless steel (ANSI 316), although the thermal conductivity of stainless steel is not as high as other metals that are used in nonfood applications. In the past, the material used for gaskets did not permit use of high temperatures. However, new heat-resistant materials for gaskets have considerably extended the use of plate heat exchangers for high-temperature applications.

To determine the rate of heat transfer across the plates, it is necessary to know the convective heat transfer coefficient on both sides of the plates. Since the mechanism is forced convection, we use a dimensionless correlation involving Nusselt number, Reynolds number, and Prandtl number. The dimensionless correlation depends upon the design of the corrugations used. An approximate expression suitable for plate heat exchanger is as follows:

$$N_{\mathrm{Nu}} = 0.4 N_{\mathrm{Re}}^{0.64} N_{\mathrm{Pr}}^{0.4} \tag{4.127}$$

To evaluate the Reynolds number, it is necessary to determine the velocity of the liquid stream in the channel. This determination is complex because of the corrugations. We will consider a simplified method to estimate the fluid velocity.

In a plate heat exchanger, the two end plates do not take part in heat transfer. Thus, the number of "thermal plates" involved in heat transfer is obtained by subtracting two from the total number of plates in a heat exchanger.

The flow rate of a liquid stream through each channel is obtained as

$$\dot{m}_{\mathrm{Hc}} = \frac{\dot{m}_{\mathrm{H}}}{\left(\dfrac{N+1}{2}\right)} \tag{4.128}$$

and

$$\dot{m}_{\mathrm{Pc}} = \frac{2\dot{m}_{\mathrm{P}}}{N+1} \tag{4.129}$$

where $\dot{m}_{\mathrm{H}}$ and $\dot{m}_{\mathrm{P}}$ are the total mass flow rates of the heating/cooling stream and the product stream (kg/s), respectively; $\dot{m}_{\mathrm{Hc}}$ and $\dot{m}_{\mathrm{Pc}}$ are the channel flow rates for heating/cooling stream and product stream (kg/s), respectively; and N is the total number of thermal plates.

The cross-sectional area of a channel between two adjacent plates is obtained as,

$$A_{\mathrm{c}} = bw \tag{4.130}$$

where b is the gap between two adjacent plates and w is the width of the plate.

The velocities of the two streams are

$$\bar{u}_{\mathrm{Pc}} = \frac{\dot{m}_{\mathrm{Pc}}}{\rho_{\mathrm{P}} A_{\mathrm{c}}} \tag{4.131}$$

$$\bar{u}_{\mathrm{Hc}} = \frac{\dot{m}_{\mathrm{Hc}}}{\rho_{\mathrm{H}} A_{\mathrm{c}}} \tag{4.132}$$

The equivalent diameter (or the hydraulic diameter), D_e, is calculated from the following expression:

$$D_e = \frac{4 \times \text{channel free-flow area for fluid stream}}{\text{wetted perimeter for the fluid}} \tag{4.133}$$

For simplicity, we use the projected area (disregarding the corrugations) to determine the wetted perimeter as

$$\text{wetted perimeter for the fluid} = 2(b + w) \tag{4.134}$$

$$\text{Channel free-flow area} = bw \tag{4.135}$$

Then,

$$D_e = \frac{4bw}{2(b + w)} \tag{4.136}$$

Since in a plate heat exchanger, $b \ll w$, we may neglect b in the denominator to obtain,

$$D_e = 2 \times b \tag{4.137}$$

The Reynolds number for each stream is obtained as

Product stream:

$$N_{Re,P} = \frac{\rho_P \bar{u}_{Pc} D_e}{\mu_P} \tag{4.138}$$

Heating/cooling stream:

$$N_{Re,H} = \frac{\rho_H \bar{u}_{Hc} D_e}{\mu_H} \tag{4.139}$$

Knowing the Reynolds number, the Nusselt number is calculated for each stream using Equation (4.127). The heat transfer coefficient is obtained as

$$h_P = \frac{N_{Nu} \times k_P}{D_e} \tag{4.140}$$

$$h_H = \frac{N_{Nu} \times k_H}{D_e} \tag{4.141}$$

where k_P and k_H are the thermal conductivities of the product and heating/cooling streams, (W/m °C), respectively.

The overall heat transfer coefficient is determined from the two values of the convective heat transfer coefficients, assuming that the conductive resistance of the thin metal plate is negligible,

$$\frac{1}{U} = \frac{1}{h_P} + \frac{1}{h_H} \tag{4.142}$$

Once the overall heat transfer coefficient is obtained using Equation (4.142), the remaining computations for the design of the plate heat exchanger are done using the effectiveness-*NTU* method similar to the procedure described in Section 4.4.7.2. For a plate heat exchanger, the relevant expressions between *NTU* and effectiveness are given in Tables 4.4 and 4.5. The calculation procedure for designing a plate heat exchanger is illustrated in the following example.

Example 4.21

A counterflow plate heat exchanger is being used to heat apple juice with hot water. The heat exchanger contains 51 plates. Each plate is 1.2 m high and 0.8 m wide. The gap between the plates is 4 mm. The heating characteristics for this heat exchanger have been previously determined to follow the following relationship, $N_{Nu} = 0.4N_{Re}^{0.64}N_{Pr}^{0.4}$. Hot water enters the heat exchanger at 95°C at a rate of 15 kg/s and apple juice enters at 15°C at a rate of 10 kg/s. It is assumed that the physical properties of water and apple juice are the same. The properties of water at 55°C are density = 985.7 kg/m^3, specific heat = 4179 J/kgK, thermal conductivity = 0.652 W/m °C, dynamic viscosity = 509.946×10^{-6} N_s/m^2, Prandtl Number = 3.27. Calculate the exit temperatures of water and apple juice.

Given

Number of plates = 51
Plate height = 1.2 m
Plate width = 0.8 m
Gap between plates = 4 mm = 0.004 m
Inlet temperature of hot water = 95°C
Mass flow rate of hot water = 15 kg/s
Inlet temperature of apple juice = 15°C
Mass flow rate of apple juice = 10 kg/s
Properties of water at (95 + 15)/2 = 55°C from Table A.4.1
Density = 985.7 kg/m^3
Specific heat = 4.179 kJ/kgK

Thermal conductivity = 0.652 W/m °C
Viscosity = 509.946×10^{-6} Ns/m^2
Prandtl number = 3.27

Approach

We will assume that the heat exchanger is operating under steady state conditions. First, we will determine the channel velocity of each fluid stream using the given flow rates and gap dimensions. Next, we will determine the Reynolds number, and using the dimensionless correlation, we will calculate Nusselt number and convective heat transfer coefficient. The convective heat transfer coefficients from both sides of a plate will be used to determine the overall heat transfer coefficient. Once the overall heat transfer coefficient is known, then we will use the effectiveness-NTU method to determine the exit temperatures of each stream.

Solution

Heat transfer coefficient on the hot water side:

1. *Equivalent diameter for the channel created between the plates is*

$$D_e = 2 \times b = 2 \times 0.004 = 0.008 \text{ m}$$

2. *Flow rate of hot water in each channel:*
Total number of plates = 51. Since the two end plates do not take part in heat exchange, the number of thermal plates = 49. Then

$$\dot{m}_{Hc} = \frac{2 \times 15}{(49 + 1)} = 0.60 \text{ kg/s}$$

3. *The cross-sectional area of each channel is*

$$A_c = 0.004 \times 0.8 = 0.0032 \text{ m}^2$$

4. *Velocity of hot water in each channel is*

$$\overline{u}_c = \frac{0.60 \text{ (kg/s)}}{985.7 \text{ (kg/m}^3) \times 0.0032 \text{ (m}^2)} = 0.19 \text{ m/s}$$

5. *Reynolds number:*

$$N_{\text{Re}} = \frac{985.7 \times 0.19 \times 0.008}{509.946 \times 10^{-6}} = 2941$$

6. *Nusselt number:*

$$N_{Nu} = 0.4 \times 2941^{0.64} \times 3.27^{0.4} = 106.6$$

Then, heat transfer coefficient on the hot water side is

$$h_H = \frac{106.6 \times 0.652}{0.008} = 8{,}688 \ W/m^2 \ ^\circ C$$

Heat transfer coefficient on the apple juice side:

7. *The flow rate of apple juice in each channel is*

$$\dot{m}_A = \frac{2 \times 10}{(49 + 1)} = 0.4 \ kg/s$$

8. *The velocity of apple juice in each channel is*

$$\overline{u}_c = \frac{0.4 \ (kg/s)}{985.7 \ (kg/m^3) \times 0.0032 \ (m^2)} = 0.127 \ m/s$$

9. *The Reynolds number is*

$$N_{Re} = \frac{985.7 \times 0.127 \times 0.008}{509.946 \times 10^{-6}} = 1961$$

10. *The Nusselt number is*

$$N_{Nu} = 0.4 \times 1961^{0.64} \times 3.27^{0.4} = 82.2$$

11. *Convective heat transfer coefficient on the apple juice side is*

$$h_H = \frac{82.2 \times 0.652}{0.008} = 6702 \ W/m^2 \ ^\circ C$$

12. *The overall heat transfer coefficient, U, is*

$$\frac{1}{U} = \frac{1}{8688} + \frac{1}{6702}$$

$$U = 3784 \ W/m^2 \ ^\circ C$$

13. *The total area of the heat exchanger is*

$$A_h = 49 \times 1.2 \times 0.8 = 47.04 \ m^2$$

14. *NTU for the heat exchanger is*

$$NTU = \frac{3784 \times 47.04}{10 \times 4179} = 4.259$$

15. *The capacity rate ratio is*

$$C^* = \frac{10}{15} = 0.67$$

16. *The exchanger effectiveness is obtained from the relationship given in Table 4.4 as*

$$\varepsilon_E = \frac{\exp[(1-0.67)\times 4.259]-1}{\exp[(1-0.67)\times 4.259]-0.67}$$

$$\varepsilon_E = 0.9$$

17. *The exit temperature of apple juice is determined by noting that* $C_{juice} = C_{min}$, *or*

$$\frac{q_{actual}}{q_{max}} = 0.9 = \frac{C_{juice}(T_{ae}-15)}{C_{min}(95-15)} = \frac{(T_{ae}-15)}{(95-15)}$$

Then, $T_{ae} = 87°C$

18. *The exit temperature of hot water is determined as*

$$0.9 = \frac{C_{water}(95-T_{we})}{C_{min}(95-15)} = 1.5\frac{(95-T_{we})}{(95-15)}$$

or, $T_{we} = 47°C$

In the given counter flow plate heat exchanger operating under steady state conditions, the hot water stream will exit at 47°C and the apple juice will be heated to 87°C.

4.4.10 Importance of Surface Characteristics in Radiative Heat Transfer

All materials in the universe emit radiation of an electromagnetic nature based on their surface temperature. At a temperature of 0 K, the emission of radiation ceases. The characteristics of the radiation are also dependent on temperature. As temperature increases, the wavelength decreases. For example, radiation emitted by the sun is shortwave compared with the longwave radiation emitted by the surface of a hot coffee mug.

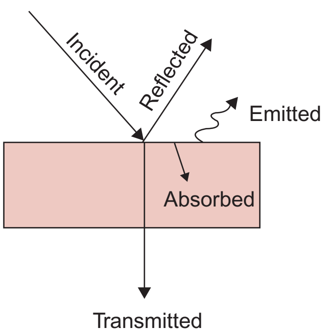


Figure 4.29 Radiant energy incident on a semi-opaque object.

When radiation of a given wavelength, as shown in Figure 4.29, is incident on an object, some of the incident radiation is reflected, some transmitted, and some absorbed. The following expression holds true:

$$\phi + \chi + \psi = 1 \tag{4.143}$$

where ϕ is absorptivity, χ is reflectivity, and ψ is transmissivity. The absorbed radiation will result in an increase of temperature.

To compare the absorption of radiation for different materials, an ideal reference called a blackbody is used. For a blackbody, the absorptivity value is 1.0. Note that nothing in the universe is a true blackbody; even lampblack has $\phi = 0.99$ and $\chi = 0.01$. Regardless, blackbody is a useful concept for comparing radiative properties of different materials.

The absolute magnitudes of ϕ, χ, and ψ depend on the nature of the incident radiation. Thus, a brick wall of a house is opaque to visible light but transparent to radio waves.

Energy emitted (also called radiated) and energy reflected must be clearly distinguished. These are quite different terms. A material, depending on its surface absorptivity value, will reflect some of the incident radiation. In addition, based on its own temperature, it will emit radiation, as shown in Figure 4.29. The amount of radiation emitted can be computed from Equation (4.20).

Kirchoff's law states that the emissivity of a body is equal to its absorptivity for the same wavelength. Thus, mathematically,

$$\varepsilon = \phi \tag{4.144}$$

This identity is discussed in Example 4.22.

Example 4.22

Compare the selection of white paint versus black paint for painting the rooftop of a warehouse. The objective is to allow minimum heat gain from the sun during summer.

Given

White paint from Table A.3.3:

$\varepsilon_{shortwave} = 0.18$

$\varepsilon_{longwave} = 0.95$

Black paint from Table A.3.3:

$\varepsilon_{shortwave} = 0.97$

$\varepsilon_{longwave} = 0.96$

Approach

From the emissivity values, we will examine the use of white or black paint for both short- and longwave radiation.

Solution

1. *White paint: $\varepsilon_{shortwave} = 0.18$, therefore from Kirchoff's law $\phi_{shortwave} = 0.18$, so $\chi_{shortwave} = 1 - 0.18$, assuming $\psi = 0$, thus $\chi_{shortwave} = 0.82$. Thus, of the total shortwave radiation incident on the rooftop, 18% is absorbed and 82% reflected.*
2. *White paint, $\varepsilon_{longwave} = 0.95$. The white-painted surface, in terms of longwave radiation, emits 95% of radiation emitted by a blackbody.*
3. *Black paint: $\varepsilon_{shortwave} = 0.97$, therefore using Kirchoff's Law $\phi_{shortwave} = 0.97$. Thus, of the total shortwave radiation incident on the rooftop, 97% of radiation incident on it is absorbed and 3% is reflected.*
4. *Black paint: $\varepsilon_{longwave} = 0.96$. The black-painted surface, in terms of longwave radiation, emits 96% of radiation emitted by a blackbody.*
5. *White paint should be selected, as it absorbs only 18% shortwave (solar) radiation compared to 97% by black paint. Both black and white paint are similar in emitting longwave radiation to the surroundings.*

4.4.11 Radiative Heat Transfer between Two Objects

The transfer of heat by radiation between two surfaces is dependent on the emissivity of the radiating surface and the absorptivity of that same surface. The expression normally used to describe this type of heat transfer is as follows:

$$q_{1-2} = A\sigma(\varepsilon_1 T_{A1}^4 - \phi_{1-2} T_{A2}^4) \tag{4.145}$$

where ε_1 is the emissivity of the radiating surface at temperature T_{A1}, and ϕ_{1-2} is the absorptivity of the surface for radiation emitted at temperature T_{A2}.

Although the basic expression describing radiative heat transfer is given by Equations (4.20) and (4.145), one of the important factors requiring attention is the shape of the object. The shape factor accounts for the fraction of the radiation emitted by the high-temperature surface that is not absorbed by the low-temperature surface. For example, Equation (4.145) assumes that all radiation emitted at temperature T_{A1} is absorbed by the surface at temperature T_{A2}. If both the surfaces are blackbodies, then the expression describing heat transfer and incorporating the shape factor would be as follows:

$$q_{1-2} = \sigma F_{1-2} A_1 (T_{A1}^4 - T_{A2}^4) \tag{4.146}$$

where F_{1-2} is the shape factor, and it physically represents the fraction of the total radiation leaving the surface A_1 that is intercepted by the surface A_2. The values of shape factors have been tabulated and presented in the form of curves of the type shown in Figures 4.30 and 4.31. In the first case, the shape factors deal only with adjacent rectangles, which are in perpendicular planes. Figure 4.31 can be utilized for various shapes, including disks, squares, and rectangles.

Equation (4.146) does not account for nonblackbodies, and Equation (4.20) does not account for the shape factor; therefore, an expression that combines the two must be used. Such an expression would be

$$q_{1-2} = \sigma A_1 \xi_{1-2}(T_{A1}^4 - T_{A2}^4) \tag{4.147}$$

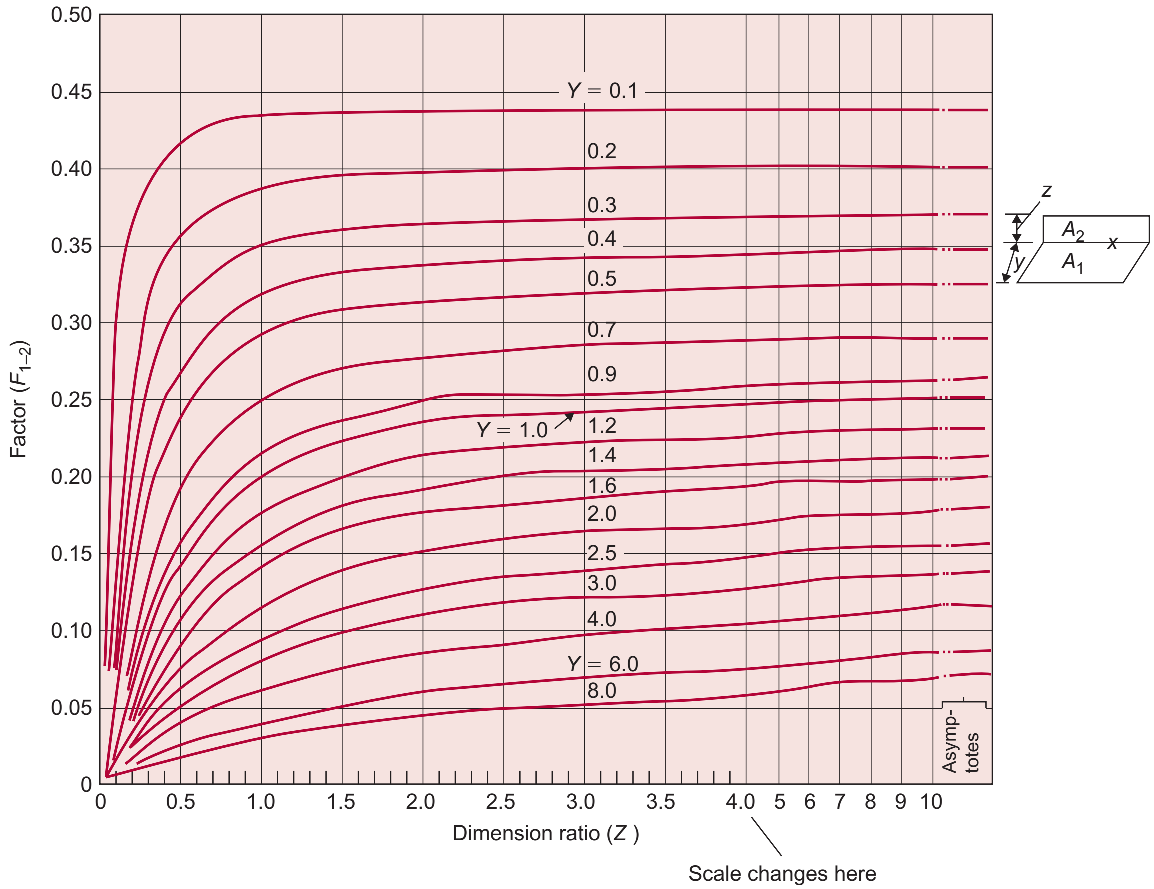


Figure 4.30 Shape factors for adjacent rectangles in perpendicular planes. Y (dimension ratio) = y/x; Z = z/x. (Adapted from Hottel, 1930)

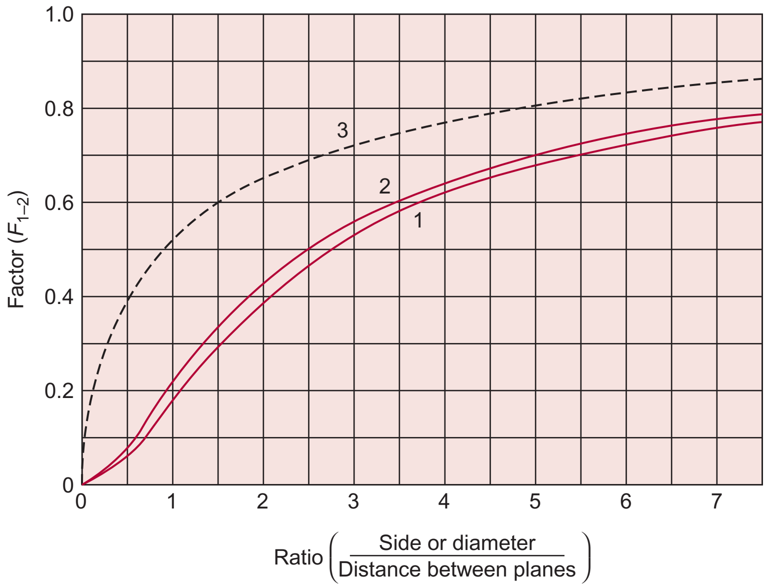


Figure 4.31 Shape factors for equal and parallel squares, rectangles, and disks: (1) direct radiation between disks; (2) direct radiation between squares; (3) total radiation between squares or disks connected by nonconducting but reradiating walls. (Adapted from Hottel, 1930)

where the ξ_{1-2} factor accounts for both shape and emissivity. This factor can be evaluated by the following expression:

$$\xi_{1-2} = \frac{1}{\frac{1}{F_{1-2}} + \left(\frac{1}{\varepsilon_1} - 1\right) + \frac{A_1}{A_2}\left(\frac{1}{\varepsilon_2} - 1\right)} \tag{4.148}$$

Equations (4.147) and (4.148) can be used to compute the net radiant heat transfer between two gray bodies in the presence of radiating surfaces at uniform temperatures.

Example 4.23

Compute the radiative heat transfer received by a rectangular product moving through a radiation-type heater (Fig. E4.20). The radiation source is one vertical wall of a heater and is held at a constant temperature of 200°C while the product is moving perpendicular to the radiation source. The product temperature is 80°C with an emissivity of 0.8. The product dimensions are 15 × 20 cm, and the radiation source is 1 × 5 m.

Given

Temperature of heater = 200°C
Product temperature = 80°C
Emissivity of product = 0.8
Product dimensions = 0.15 m × 0.2 m
Heater dimensions = 1 m × 5 m

Approach

To compute the ξ_{1-2} factor from Equation (4.148), we will use Figure 4.30, for objects perpendicular to each other, for F_{1-2} value. Then we will use Equation (4.147) to calculate radiative heat received by the rectangular product.

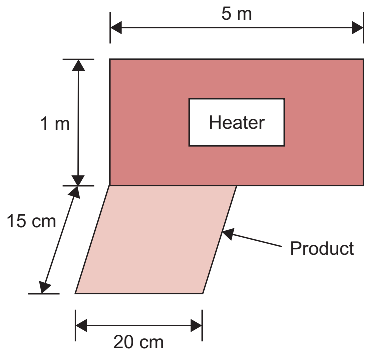


Figure E4.20 Radiation heater.

Solution

1. *To use Equation (4.147), the ξ factor must be computed from Equation (4.148).*

$$\xi_{1-2} = \frac{1}{\dfrac{1}{F_{1-2}} + \left(\dfrac{1}{1} - 1\right) + \dfrac{5}{0.03}\left(\dfrac{1}{0.8} - 1\right)}$$

where F_{1-2} must be obtained from Figure 4.30 for $z/x = 5.0$ and $y/x = 0.75$.

$$F_{1-2} = 0.28$$

Then

$$\xi_{1-2} = \frac{1}{(3.57 + 0 + 41.67)} = \frac{1}{45.24} = 0.0221$$

2. *From Equation (4.147),*

$$\begin{aligned} q_{1-2} &= (5.669 \times 10^{-8}\ \mathrm{W/[m^2\ K^4]})(0.0221)(5\ \mathrm{m^2}) \\ &\quad \times [(473\ \mathrm{K})^4 - (353\ \mathrm{K})^4] \\ &= 216\ \mathrm{W} \end{aligned}$$

4.5 UNSTEADY-STATE HEAT TRANSFER

Unsteady-state (or transient) heat transfer is that phase of the heating and cooling process when the temperature changes as a function of both location and time. By contrast, in steady-state heat transfer, temperature varies only with location. In the initial unsteady-state period, many important reactions in the food may take place. With thermal processes, the unsteady-state phase may even dominate the entire process; for example, in several pasteurization and food-sterilization processes, the unsteady-state period is an important component of the process. Analysis of temperature variations with time during the unsteady-state period is essential in designing such a process.

Since temperature is a function of two independent variables, time and location, the following partial differential equation is the governing equation for a one-dimensional case:

$$\frac{\partial T}{\partial t} = \frac{k}{\rho c_p r^n}\frac{\partial}{\partial r}\left(r^n \frac{\partial T}{\partial r}\right) \tag{4.149}$$

where T is temperature (°C), t is time (s), and r is distance from center location (m). We can make this equation specific for a particular geometrical shape using $n = 0$ for a slab, $n = 1$ for a cylinder, and $n = 2$ for a sphere. The combination of properties $k/\rho c_p$ is defined as thermal diffusivity, α. If the rate of heat transfer at the surface of the object is due to convection, then

$$k\frac{\partial T}{\partial r}\Big|_{r=R} = h(T_a - T_s) \tag{4.150}$$

where h is the convective heat transfer coefficient (W/[m^2 °C]), T_a is the temperature of heating or cooling medium far away from the surface (°C), and T_s is the temperature at the surface (°C).

The procedure we use to solve the governing equation, Equation (4.149), involves the use of advanced mathematics, which is beyond the scope of this book. Myers (1971) gives the complete derivation for various types of boundary-value problems encountered in unsteady-state heat transfer. Due to mathematical complexity, the analytical solution of Equation (4.149) is possible only for objects of simplified geometrical shapes, such as a sphere, an infinite cylinder, or an infinite slab.

An infinite cylinder is a "long" cylinder of radius r, a sphere is of radius r, an infinite plate is a "large" plane wall of thickness $2z$. These three objects are geometrically and thermally symmetric about their centerline (for a cylinder), center plane (for a slab), or center point (for a sphere), as shown in Figure 4.32.

Consider heating an infinitely long cylindrical object. Initially, the object is assumed to have a uniform temperature, T_i. At time $t = 0$, we place this object in a heating medium maintained at a constant temperature T_a. A constant heat transfer coefficient, h, describes the convective heat transfer at the surface of the object. Temperature profiles at different time intervals inside the object are shown in Figure 4.33. At time $t = 0$, the temperature is uniform at T_i. At time, $t = t_1$, the temperature along the wall increases, establishing a temperature gradient within the object that promotes heat conduction. At time $t = t_2$, the temperature at the center is still at T_i. However, with the passage of time, at time $t = t_3$, the centerline temperature begins to increase, and eventually at $t = t_4$ the temperature of the cylinder becomes uniform at T_a. At this time, the cylinder is in thermal equilibrium with the surrounding medium, and heat transfer ceases. Note that there is no heat transfer from the axial ends of the cylinder.

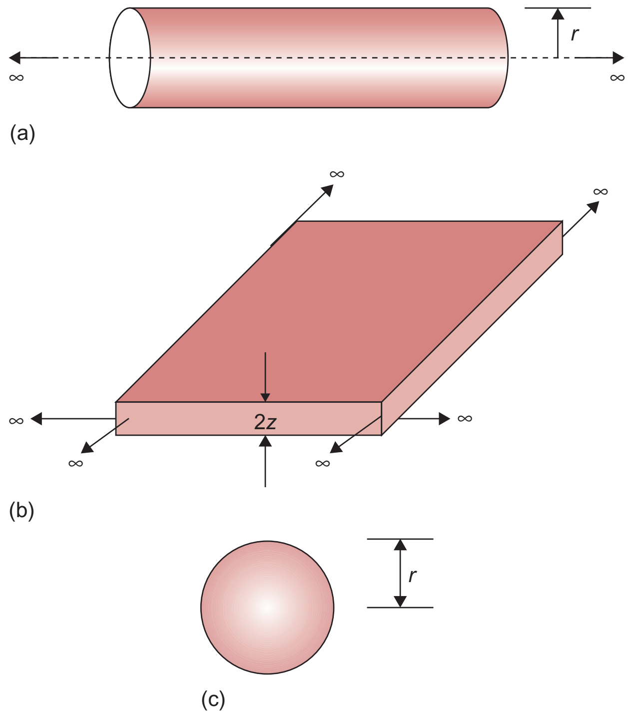
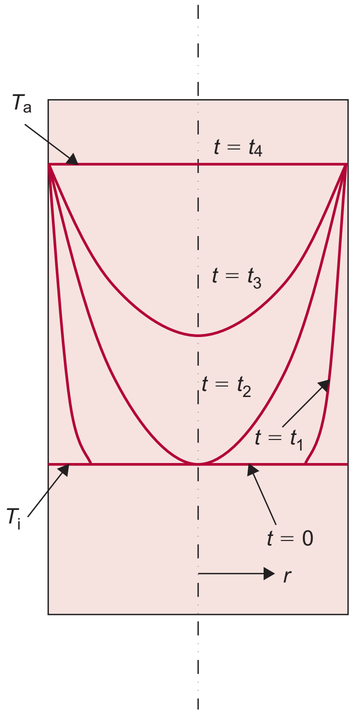


Figure 4.32 (a) An infinite cylinder, (b) an infinite plate, (c) a sphere.



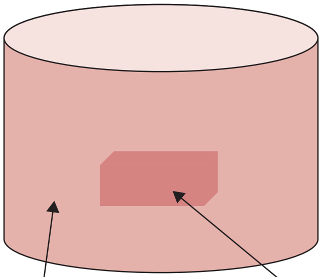
W

Figure 4.33 Temperature profiles as a function of time in an infinitely long cylinder.

Because the cylinder is infinitely long, the implication of "infinite" length in this case is that the heat transfer is only in the radial direction and not axial. Similarly, for an infinite slab of thickness, $2z$, the heat transfer is only from the two faces around the thickness $2z$ of the slab. The slab extends to infinity along the other four faces, and no heat transfer takes place through those sides. We will explore these concepts in greater detail in the following sections.

4.5.1 Importance of External versus Internal Resistance to Heat Transfer

In transient heat transfer analysis, one of the first steps is to consider the relative importance of heat transfer at the surface and interior of an object undergoing heating or cooling. Consider an object that is suddenly immersed in a fluid (Fig. 4.34). If the fluid is at a temperature different from the initial temperature of the solid, the temperature inside the solid will increase or decrease until it reaches a value in equilibrium with the temperature of the fluid.



■ **Figure 4.34** A solid object suddenly exposed to a heating/cooling medium.

During the unsteady-state heating period, the temperature inside the solid object (initially at a uniform temperature) will vary with location and time. Upon immersing the solid in the fluid, the heat transfer from the fluid to the center of the solid will encounter two resistances: Convective resistance in the fluid layer surrounding the solid, and conductive resistance inside the solid. The ratio of the internal resistance to heat transfer in the solid to the external resistance to heat transfer in the fluid is defined as the B_{iot} number, N_{Bi}.

$$\frac{\text{internal conductive resistance within the body}}{\text{external convective resistance at the surface of the body}} = N_{Bi} \tag{4.151}$$

or

$$N_{Bi} = \frac{d_c/k}{1/h} \tag{4.152}$$

or

$$N_{Bi} = \frac{hd_c}{k} \tag{4.153}$$

where d_c is a characteristic dimension.

According to Equation (4.151), if the convective resistance at the surface of a body is much smaller than the internal conductive resistance, then the Biot number will be high. For Biot numbers greater than 40, there is negligible surface resistance to heat transfer. On the other hand, if internal conductive resistance to heat transfer is small, then the Biot number will be low. For Biot numbers less than 0.1, there is negligible internal resistance to heat transfer. Between a Biot number of 0.1 and 40, there is a finite internal and external resistance to heat transfer. Steam condensing on the surface of a broccoli stem will result in negligible surface resistance to heat transfer ($N_{Bi} \geq 40$). On the other hand, a metal can containing hot tomato paste, being cooled in a stream of cold air, will present finite internal and surface resistance to heat transfer, and a small copper sphere placed in stagnant heated air will have a Biot number of less than 0.1. In the following subsections, we will consider these three cases separately.

4.5.2 Negligible Internal Resistance to Heat Transfer ($N_{Bi} < 0.1$)—A Lumped System Analysis

For a Biot number smaller than 0.1, there is a negligible internal resistance to heat transfer. This condition will occur in heating and cooling of most solid metal objects but not with solid foods, because the thermal conductivity of a solid food is relatively low.

Negligible internal resistance to heat transfer also means that the temperature is nearly uniform throughout the interior of the object. For this reason, this case is also referred to as a "lumped" system. This condition is obtained in objects with high thermal conductivity when they are placed in a medium that is a poor conductor of heat, such as motionless air. In these cases heat is transferred instantaneously into the object, thus avoiding temperature gradients with location. Another way to obtain such a condition is a well-stirred liquid food in a container. For this special case, there will be no temperature gradient with location, as the product is well mixed.

A mathematical expression to describe heat transfer for a negligible internal resistance case may be developed as follows.

Consider an object at low uniform temperature T_i, immersed in a hot fluid at temperature T_a, as shown in Figure 4.34. During the unsteady-state period, a heat balance across the system boundary gives

$$q = \rho c_p V \frac{dT}{dt} = hA(T_a - T) \tag{4.154}$$

where T_a is the temperature of the surrounding medium, and A is the surface area of the object.

By separating variables,

$$\frac{dT}{(T_a - T)} = \frac{hA\,dt}{\rho c_p V} \tag{4.155}$$

Integrating, and setting up limits,

$$\int_{T_i}^{T} \frac{dT}{T_a - T} = \frac{hA}{\rho c_p V} \int_0^t dt \tag{4.156}$$

$$-\ln(T_a - T)|_{T_i}^{T} = \frac{hA}{\rho c_p V}(t - 0) \tag{4.157}$$

$$-\ln\left(\frac{T_a - T}{T_a - T_i}\right) = \frac{hAt}{\rho c_p V} \tag{4.158}$$

Rearranging terms,

$$\frac{T_a - T}{T_a - T_i} = e^{-(hA/\rho c_p V)t} \tag{4.159}$$

Rewriting Equation (4.159) as

$$\frac{T_a - T}{T_a - T_i} = e^{-bt} \tag{4.160}$$

where

$$b = \frac{hA}{\rho c_p V}$$

In Equation (4.160), the numerator $T_a - T$ on the left-hand side is the unaccomplished temperature difference between the heat transfer medium and the object. The denominator is the maximum temperature difference at the start of the heating/cooling process. Thus, the temperature ratio shown on the left-hand side of this equation is the unaccomplished temperature fraction. At the start of the heating/cooling process, the unaccomplished temperature fraction is one, and it decreases with time. The right-hand side of Equation (4.160) shows an exponentially decreasing (or decaying) function. This implies that with increasing passage of time the unaccomplished temperature fraction decreases but it never reaches a value of zero, it approaches zero asymptotically. Furthermore, when an object is being heated, with a higher value of b, the object temperature increases more rapidly (with a greater decay in temperature difference). The b value is influenced directly by the convective conditions on the surface described by h, its thermal properties, and the size. Small objects with low specific heat take a shorter time to heat or cool.

Example 4.24

Calculate the temperature of tomato juice (density = 980 kg/m^3) in a steam-jacketed hemispherical kettle after 5 minutes of heating (see Fig. E4.21). The radius of the kettle is 0.5 m. The convective heat-transfer coefficient in the steam jacket is 5000 W/(m^2 °C). The inside surface temperature of the kettle is 90°C. The initial temperature of tomato juice is 20°C. Assume specific heat of tomato juice is 3.95 kJ/(kg °C).

Given

Kettle:

Surface temperature $T_a = 90°C$

Radius of kettle = 0.5 m

Tomato juice:

Initial temperature $T_i = 20°C$

Specific heat $c_p = 3.95$ kJ/(kg °C)

Density $\rho = 980$ kg/m^3

Time of heating $t = 5$ min

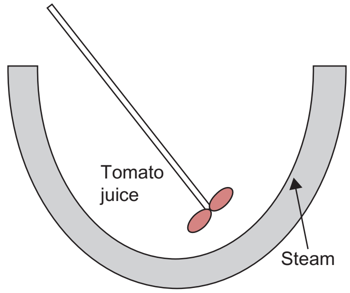


Figure E4.21 Heating tomato juice in a hemispherical steam jacketed kettle.

Approach

Since the product is well mixed, there are no temperature gradients inside the vessel, and we have negligible internal resistance to heat transfer. We will use Equation (4.159) to find the temperature after 5 min.

Solution

1. We will use Equation (4.159). First, the inside surface area and volume of the hemispherical kettle are computed:

$$A = 2\pi r^2 = 2\pi(0.5)^2 = 1.57\ m^2$$

$$V = \frac{2}{3}\pi r^3 = \frac{2}{3}\pi(0.5)^3 = 0.26\ m^3$$

2. Using Equation (4.159) :

$$\frac{90 - T}{90 - 20} = \exp\frac{-(5000\ W/[m^2\ °C])(1.57\ m^2)(300\ s)}{(980\ kg/m^3)(3.95\ kJ/[kg\ °C])(1000\ J/kJ)(0.26\ m^3)}$$

$$\frac{90 - T}{90 - 20} = 0.096$$

$$T = 83.3°C$$

3. The product temperature will rise to 83.3°C in 5 min of heating.

Example 4.25

An experiment was conducted to determine surface convective heat-transfer coefficient for peas being frozen in an air-blast freezer. For this purpose, a metal analog of peas was used. The analog was a solid copper ball with a diameter of 1 cm. A small hole was drilled to the center of the copper ball, and a thermocouple junction was located at the center using a high-conductivity epoxy. The density of copper is 8954 kg/m^3, and its specific heat is 3830 J/(kg K). The copper ball (at a uniform initial temperature of 10°C) was hung in the path of air flow (at −40°C) and the center temperature indicated by the thermocouple was recorded. The following table lists the temperature at 1-min intervals for 14 min. Determine the surface heat-transfer coefficient from these data.

Time (s)	Temperature (°C)
0	10.00
60	9.00
120	8.00
180	7.00
240	6.00
300	5.00
360	4.00
420	3.50
480	2.50
540	1.00
600	1.00
660	0.00
720	−2.00
780	−2.00
840	−3.00

Given

Diameter of copper ball $D = 1$ *cm*
Density of copper $\rho = 8954$ *kg/m*^{3}
Specific heat of copper $c_p = 3830$ *J/(kg K)*
Initial temperature of copper $T_i = 10°C$
Temperature of cold air $= -40°C$

Approach

We will use a modified form of Equation (4.159) to plot the temperature–time data. If these data are plotted on a semilog paper, we can obtain the h value from the slope. An alternative approach is to use statistical software to develop a correlation and determine the slope.

Solution

1. *Equation (4.159) may be rewritten as follows:*

$$ln(T - T_a) = ln(T_i - T_a) - \frac{hAt}{\rho c_p V}$$

2. *The tabulated data for temperature and time is converted to* $ln(T - T_a)$.

3. *Using a statistical software (e.g., StatView™), a correlation is obtained between t and ln(T−T_a). The results are*

$$slope = -3.5595 \times 10^{-4}\ l/s$$

4. *Thus,*

$$\frac{hA}{\rho c_p V} = 3.5595 \times 10^{-4}$$

5. *Surface area of sphere,* $A = 4\pi r^2$
   Volume of sphere, $V = 4\pi r^3/3$
6. *Substituting given and calculated values in the expression in step (4), we get*

$$h = 20\ W/(m^2\ °C)$$

7. *A pea held in the same place as the copper ball in the blast of air will experience a convective heat-transfer of 20 W/(m^2 °C).*

4.5.3 Finite Internal and Surface Resistance to Heat Transfer ($0.1 \le N_{Bi} \le 40$)

As noted previously, the solution of Equation (4.149) is complicated, and it is available only for well-defined shapes such as sphere, infinite cylinder, and infinite slab. In each case, the solution is an infinite series containing trigonometric and/or transcendental functions. These solutions are as follows.

Sphere:

$$\frac{T_a - T}{T_a - T_i} = 4\left(\frac{d_c}{r}\right)\sum_{n=1}^{\infty}\frac{\sin \lambda_n - \lambda_n \cos \lambda_n}{2\lambda_n - \sin 2\lambda_n} e^{-\lambda_n^2 N_{Fo}} \sin\left(\lambda_n \frac{r}{d_c}\right) \tag{4.161}$$

With root equation,

$$1 - \lambda_n \cot\ \lambda_n = N_{Bi} \tag{4.162}$$

Infinite Cylinder:

$$\frac{T_a - T}{T_a - T_i} = 2\sum_{n=1}^{\infty}\frac{1}{\lambda_n}\frac{J_1(\lambda_n)}{J_0^2(\lambda_n) + J_1^2(\lambda_n)} e^{-\lambda_n^2 N_{Fo}} J_0\left(\lambda_n \frac{r}{d_c}\right) \tag{4.163}$$

with root equation,

$$\lambda_n \frac{J_1(\lambda_n)}{J_0(\lambda_n)} = N_{Bi} \tag{4.164}$$

Infinite Slab:

$$\frac{T_a - T}{T_a - T_i} = 4\sum_{n=1}^{\infty}\left(\frac{\sin \lambda_n}{2\lambda_n + \sin 2\lambda_n}\right)e^{-\lambda_n^2 N_{Fo}} \cos\left(\lambda_n \frac{x}{d_c}\right) \tag{4.165}$$

With root equation,

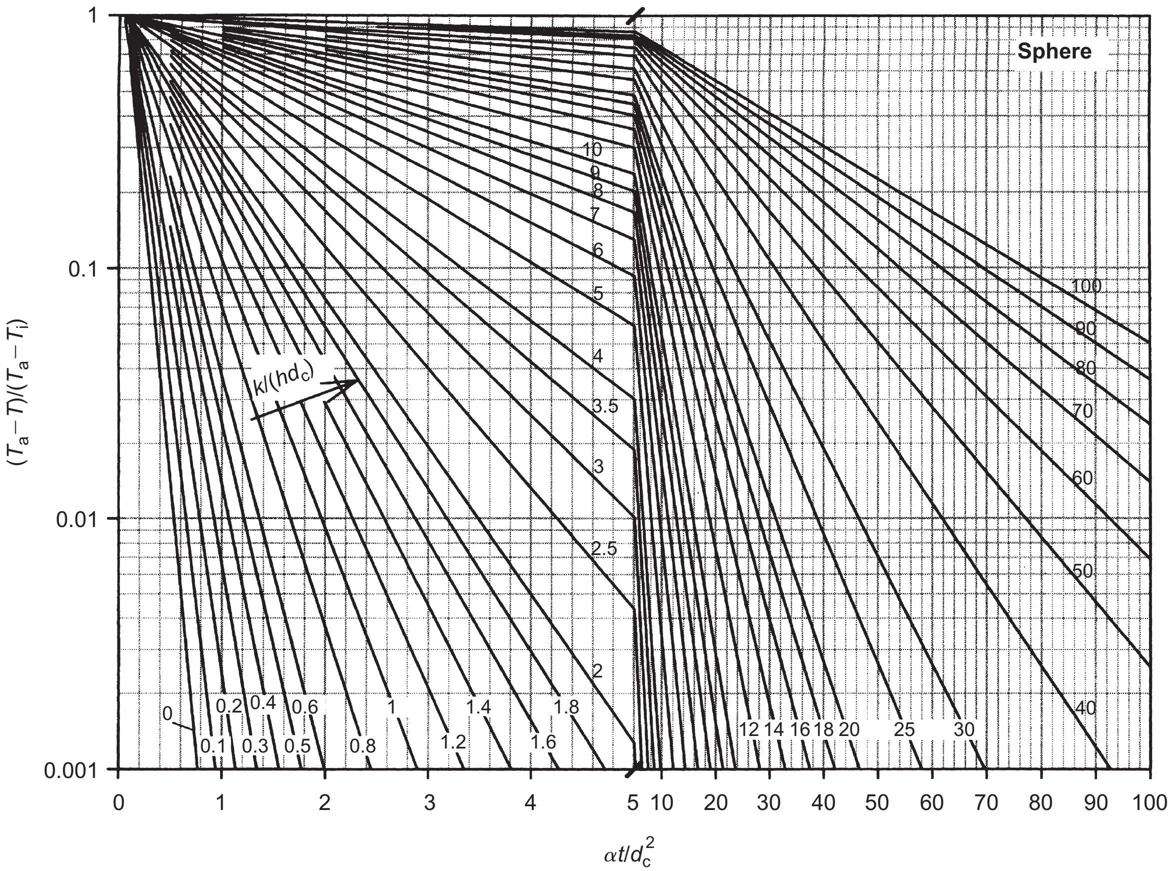
$$\lambda_n \tan \lambda_n = N_{Bi} \tag{4.166}$$

These analytical solutions with infinite series may be programmed into a spreadsheet for use on a computer. We will illustrate this procedure later in this section with an example. These solutions have also been reduced to simple temperature–time charts that are relatively easy to use. In constructing a temperature–time chart for a typical transient heat transfer problem, the variety of factors can be numerous: for example, r, t, k, p, c_p, h, T_i, and T_a. However, these factors may be combined into three dimensionless numbers, making it convenient to develop charts for universal use regardless of units used for measuring these factors. Temperature–time charts developed for the three geometric shapes (sphere, infinite cylinder, and infinite slab) are presented in Figures 4.35, 4.36, and 4.37. These charts are called Heisler charts, based on the work of Heisler (1944). The three dimensionless numbers shown on these charts are the unaccomplished temperature fraction, $(T_a - T)/(T_a - T_i)$, Biot number, N_{Bi}, and a dimensionless time expressed as the Fourier[2] number. The Fourier number is defined as follows:

$$\text{Fourier number} = N_{Fo} = \frac{k}{\rho c_p}\frac{t}{d_c^2} = \frac{\alpha t}{d_c^2} \tag{4.167}$$

where d_c is a characteristic dimension. The value for d_c indicates the shortest distance from the surface to the center of the object. The characteristic dimension for both a sphere and an infinite cylinder is the radius; for an infinite slab, it is half the thickness of the slab.

[2] Joseph Baron Fourier (1768–1830) was a French mathematician and highly regarded Egyptologist. In 1798, he accompanied Napoleon to Egypt and conducted extensive research on Egyptian antiques. From 1798 to 1801 he served as a Secretary of Institut d'Egypte in Cairo. His work Theorie Analytique de la Chaleur (The Analytical Theory of Heat) was started in 1807 in Grenoble and was completed in 1822 in Paris. He developed a mathematical basis for the conductive heat transfer in solids.



W

Figure 4.35 Temperature at the geometric center of a sphere of radius d_c.

We can examine the physical significance of the Fourier number, N_{Fo}, by rearranging Equation (4.167) as follows:

$$N_{Fo} = \frac{\alpha t}{d_c^2} = \frac{k(1/d_c)d_c^2}{\rho c_p d_c^3/t}$$

$$= \frac{\text{rate of heat conduction across } d_c \text{ in a body of volume } d_c^3 \text{ (W/°C)}}{\text{rate of heat storage in a body of volume } d_c^3 \text{(W/°C)}}$$

For a given volume element, the Fourier number is a measure of the rate of heat conduction per unit rate of heat storage. Thus, a larger value of Fourier number indicates deep penetration of heat into the solid in a given period of time.

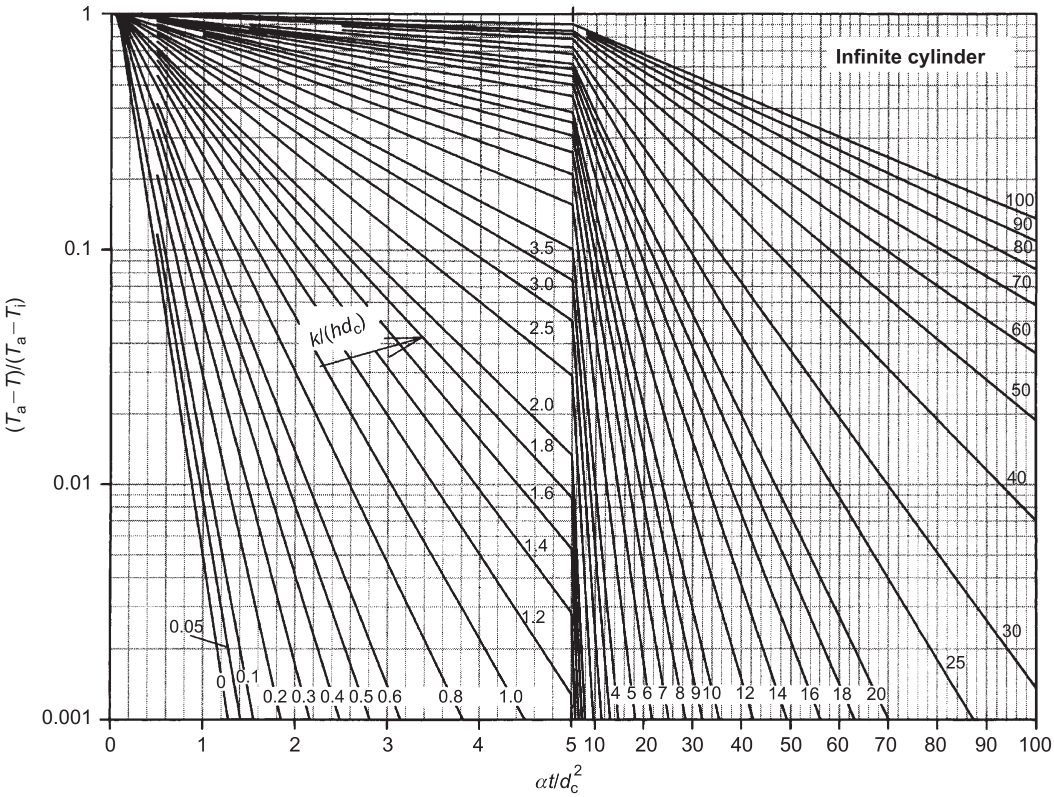


Figure 4.36 Temperature at the geometric center of an infinitely long cylinder of radius d_c.

Note that the Heisler charts shown in Figures 4.35, 4.36, and 4.37 are plotted on a log-linear scale.

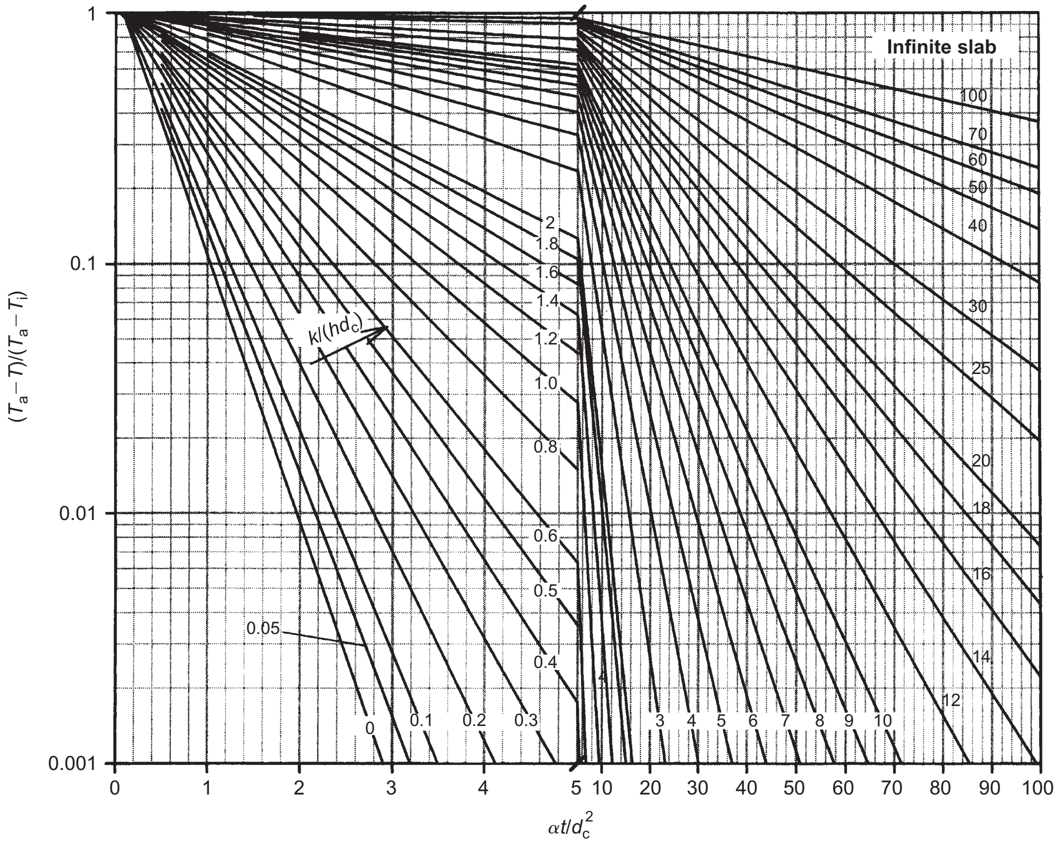
4.5.4 Negligible Surface Resistance to Heat Transfer ($N_{Bi} \geq 40$)

For situations where the Biot number is greater than 40, indicating negligible surface resistance to heat transfer, we can use the charts in Figures 4.35, 4.36, and 4.37. In these figures, the lines for $k/hd_c = 0$ represent negligible surface resistance to heat transfer.

4.5.5 Finite Objects

Myers (1971) has shown mathematically that

$$\left(\frac{T_a - T}{T_a - T_i}\right)_{\text{finite cylinder}} = \left(\frac{T_a - T}{T_a - T_i}\right)_{\text{infinite cylinder}} \times \left(\frac{T_a - T}{T_a - T_i}\right)_{\text{infinite slab}} \quad (4.168)$$

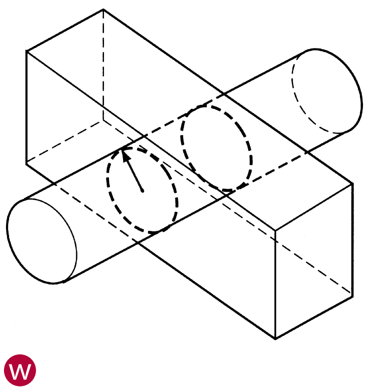


■ **Figure 4.37** Temperature at the midplane of an infinite slab of thickness $2d_c$.

and

$$\left(\frac{T_a - T}{T_a - T_i}\right)_{\text{finite brick shape}} = \left(\frac{T_a - T}{T_a - T_i}\right)_{\text{infinite slab, width}} \times \left(\frac{T_a - T}{T_a - T_i}\right)_{\text{infinite slab, depth}} \times \left(\frac{T_a - T}{T_a - T_i}\right)_{\text{infinite slab, height}} \tag{4.169}$$

These expressions allow us to determine temperature ratios for objects of finite geometry, such as a cylindrical can commonly used in heat sterilization of food. Although the mathematics to prove Equations (4.168) and (4.169) is beyond the scope of this text, Figure 4.38 may be studied as a visual aid.



■ **Figure 4.38** A finite cylinder considered as part of an infinite cylinder and an infinite slab.

A finite cylinder (Fig. 4.38) can be visualized as a part of an infinite cylinder and a part of an infinite slab. Heat transfer in a radial direction is similar to heat transfer for an infinite cylinder. Invoking the infinite cylinder shape, we mean that heat transfer to the geometric center is only through the radial direction—through the circumferential area of the cylinder—whereas the ends of the cylinder are too far to have any measurable influence on heat transfer. Heat transfer from the two end surfaces is similar to heat transfer for an infinite slab. Considering the finite cylinder to be an infinite slab will account for all the heat transfer from the two ends of the cylindrical can while ignoring heat transfer in the radial direction. This approach allows us to include heat transfer both from the radial direction and from the cylinder ends. Similarly, a brick-shape object can be considered to be constructed from three infinite slabs maintaining width, depth, and height, respectively, as the finite thickness.

4.5.6 Procedures to Use Temperature–Time Charts

The following steps may be used to determine heat transfer in finite objects with the use of temperature–time charts.

Heat transfer to an object of finite cylindrical shape, such as a cylindrical can, requires the use of temperature–time charts for both infinite cylinder and infinite slab. Thus, if the temperature at the geometric center of the finite cylinder is required at a given time, the following steps may be used.

For an infinite cylinder:

1. Calculate the Fourier number, using the radius of the cylinder as the characteristic dimension.
2. Calculate the Biot number, using the radius of the cylinder as the characteristic dimension. Calculate the inverse of Biot number for use in the Heisler Charts.
3. Use Figure 4.36 to find the temperature ratio.

For an infinite slab:

1. Calculate the Fourier number, using the half-height as the characteristic dimension.
2. Calculate the Biot number, using the half-height as the characteristic dimension. Calculate the inverse of Biot number for use in the Heisler Charts.
3. Use Figure 4.37 to determine the temperature ratio.

The temperature ratio for a finite cylinder is then calculated from Equation (4.168). We can compute the temperature at the geometric center if the surrounding medium temperature T_a and initial temperature T_i are known.

Steps similar to the preceding may be used to compute temperature at the geometric center for a finite slab-shaped object (such as a parallelepiped or a cube). In the case of a spherical object such as an orange, Figure 4.35 for a sphere is used.

A major drawback of the Heisler charts is that they are difficult to use for situations when the Fourier number is small. For example, with problems involving transient heat transfer in foods, the Fourier numbers are often less than 1 because of the low thermal diffusivity of foods. In these cases, charts with expanded scales (see Appendix A.8), based on the work of Schneider (1963), are helpful. The procedure to use these expanded-scale charts is exactly the same as with the Heisler charts. Note that the expanded-scale charts use Biot numbers directly, without inverting them, and they are plotted on a linear-log scale, whereas the Heisler charts are plotted on a log-linear scale.

Examples 4.26 to 4.28 illustrate the procedures presented in this section.

Example 4.26

Estimate the time when the temperature at the geometric center of a 6 cm diameter apple held in a 2°C water stream reaches 3°C. The initial uniform temperature of the apple is 15°C. The convective heat transfer coefficient in water surrounding the apple is 50 W/(m^2 °C). The properties of the apple are thermal conductivity $k = 0.355$ W/(m °C); specific heat $c_p = 3.6$ kJ/(kg °C); and density $\rho = 820$ kg/m^3.

Given

Diameter of apple = 0.06 m
Convective heat-transfer coefficient $h = 50$ W/(m^2 °C)
Temperature of water stream $T_a = 2$°C
Initial temperature of apple $T_i = 15$°C
Final temperature of geometric center $T = 3$°C
Thermal conductivity $k = 0.355$ W/(m °C)
Specific heat $c_p = 3.6$ kJ/(kg °C)
Density $\rho = 820$ kg/m^3

Approach

Considering an apple to be a sphere in shape, we will use Figure 4.35 to find the Fourier number. The time of cooling will be computed from Equation (4.167).

Solution

1. *From given temperatures, we first calculate the temperature ratio.*

$$\left(\frac{T_a - T}{T_a - T_i}\right) = \frac{2 - 3}{2 - 15} = 0.077$$

2. *The Biot number is computed as*

$$N_{Bi} = \frac{hd_c}{k} = \frac{(50\ W/[m^2\ °C])(0.03\ m)}{(0.355\ W/[m\ °C])} = 4.23$$

Thus,

$$\frac{1}{N_{Bi}} = 0.237$$

3. *From Figure 4.35, for a temperature ratio of 0.077 and ($1/N_{Bi}$) of 0.237, the Fourier number can be read as*

$$N_{Fo} = 0.5$$

4. *The time is calculated from the Fourier number.*

$$\frac{k}{\rho c_p}\frac{t}{d_c^2} = 0.5$$

$$t = \frac{(0.5)(820\ kg/m^3)(3.6\ kJ/[kg\ °C])(0.03\ m)^2(1000\ J/kJ)}{(0.355\ W/[m\ °C])}$$

$$= 3742\ s$$

$$= 1.04\ h$$

Example 4.27

Estimate the temperature at the geometric center of a food product contained in a 303 × 406 can exposed to boiling water at 100°C for 30 minutes. The product is assumed to heat and cool by conduction. The initial uniform temperature of the product is 35°C. The properties of the food are thermal conductivity $k = 0.34$ W/(m °C); specific heat $cp = 3.5$ kJ/(kg °C); and density

$\rho = 900$ kg/m^3. The convective heat transfer coefficient for the boiling water is estimated to be 2000 W/(m^2 °C).

Given

Can dimensions:

$$\textit{Diameter} = 3\tfrac{3}{16}\ \textit{inches} = 0.081\ m$$

$$\textit{Height} = 4\tfrac{6}{16}\ \textit{inches} = 0.11\ m$$

Convective heat transfer coefficient $h = 2000$ *W/(m^2 °C)*
Temperature of heating media $T_a = 100°C$
Initial temperature of food $T_i = 35°C$
Time of heating = 30 min = 1800 s
Properties:

$$k = 0.34\ W/(m\ °C)$$

$$c_p = 3.5\ kJ/(kg\ °C)$$

$$\rho = 900\ kg/m^3$$

Approach

Since a finite cylindrical can may be considered a combination of infinite cylinder and infinite slab, we will use time–temperature figures for both these shapes to find respective temperature ratios. The temperature ratio for a finite cylinder will then be calculated from Equation (4.168).

Solution

1. *First, we estimate temperature ratio for an infinite cylinder.*
2. *Biot number* $= hd_c/k$ *where* d_c *is radius* $= 0.081/2 = 0.0405$ *m*

$$N_{Bi} = \frac{(2000\ W/[m^2\ °C])(0.0405\ m)}{(0.34\ W/[m\ °C])} = 238$$

Thus,

$$\frac{1}{N_{Bi}} = 0.004$$

3. *The Fourier number for an infinite cylinder is*

$$N_{Fo} = \frac{k}{\rho c_p}\left(\frac{t}{d_c^2}\right)$$

$$= \frac{(0.34\ \mathrm{W}/[\mathrm{m}\ ^\circ\mathrm{C}])(1800\ \mathrm{s})}{(900\ \mathrm{kg}/\mathrm{m}^3)(3.5\ \mathrm{kJ}/[\mathrm{kg}\ ^\circ\mathrm{C}])(1000\ \mathrm{J}/\mathrm{kJ})(0.0405\ \mathrm{m})^2}$$

$$= 0.118$$

4. *The temperature ratio can be estimated from Figure 4.36 for* $1/N_{Bi} = 0.004$ *and* $N_{Fo} = 0.118$ *as*

$$\left(\frac{T_a - T}{T_a - T_i}\right)_{\text{infinite cylinder}} = 0.8$$

5. *Next, we estimate the temperature ratio for an infinite slab.*
6. *Biot number* $= hd_c/k$ *where* d_c *is half-height* $= 0.11/2 = 0.055$ m

$$N_{Bi} = \frac{(2000\ \mathrm{W}/[\mathrm{m}^2\ ^\circ\mathrm{C}])(0.055\ \mathrm{m})}{(0.34\ \mathrm{W}/[\mathrm{m}\ ^\circ\mathrm{C}])} = 323.5$$

Thus,

$$1/N_{Bi} = 0.003$$

7. *The Fourier number for an infinite slab is*

$$N_{Fo} = \frac{kt}{\rho c_p d_c^2}$$

$$= \frac{(0.34\ \mathrm{W}/[\mathrm{m}\ ^\circ\mathrm{C}])(1800\ \mathrm{s})}{(900\ \mathrm{kg}/\mathrm{m}^3)(3.5\ \mathrm{kJ}/[\mathrm{kg}\ ^\circ\mathrm{C}])(1000\ \mathrm{J}/\mathrm{kJ})(0.055\ \mathrm{m})^2}$$

$$= 0.064$$

8. *The temperature ratio can be estimated from Figure 4.37 for* $1/N_{Bi} = 0.003$ *and* $N_{Fo} = 0.064$ *as*

$$\left(\frac{T_a - T}{T_a - T_i}\right)_{\text{infinite slab}} = 0.99$$

9. *The temperature ratio for a finite cylinder is computed using Equation (4.168)*

$$\left(\frac{T_a - T}{T_a - T_i}\right)_{\text{finite cylinder}} = (0.8)(0.99) = 0.792$$

Therefore,

$$\begin{aligned} T &= T_a - 0.792(T_a - T_i) \\ &= 100 - 0.792(100 - 35) \\ &= 48.4°C \end{aligned}$$

10. *The temperature at the geometric center of the can after 30 minutes. will be 48.4°C. Note that most of the heat transfers radially; only a small amount of heat transfers axially, since*

$$\left(\frac{T_a - T}{T_a - T_i}\right)_{\text{infinite slab}} = 0.99$$

or a value close to 1. If

$$\left(\frac{T_a - T}{T_a - T_i}\right) = 1$$

then $T = T_i$; this means that the temperature at the end of the heating period is still T_i, the initial temperature, indicating no transfer of heat. Conversely, if

$$\left(\frac{T_a - T}{T_a - T_i}\right) = 0$$

then $T = T_a$, indicating that the temperature of the end-of-heating period equals that of the surrounding temperature.

Example 4.28

Using Equations (4.161), (4.163), and (4.165), develop spreadsheet programs, and compare the calculated results with the values obtained from charts given in Figures 4.35, 4.36, and 4.37.

Approach

We will use spreadsheet EXCEL and program modified forms of Equations (4.161), (4.163), and (4.165) for negligible surface resistance to heat transfer ($N_{Bi} > 40$). Since these equations involve series solutions, we will consider the first 30 terms of each series, which should be sufficiently accurate for our purposes.

Solution

The spreadsheets written for a sphere, infinite cylinder, and infinite slab are shown in Figures E4.22, E4.23, and E4.24, respectively.

	A	B	C	D	E	F	G	H
1								
2		SPHERE - Negligible surface resistance to heat transfer						
3								
4								
5		Domain: 0 < r < d_c where d_c is characteristic dimension, or radius of sphere)						
6								
7						Terms of series	=((−1^(E9+1))/E9*EXP (−E9*E9*PI()*PI()*$C$8)* SIN(E9*PI()*$C$9))	
8		Fourier number	0.3		n	term_n		
9		r/d_c	0.00001		1	1.6265E−06		
10					2	−2.2572E−10		
11		Temperature ratio	0.104		3	8.3965E−17		
12					4	−8.3722E−26		
13		=SUM(F9:F38)*2/PI()*(1/$C$9)			5	2.2376E−37		
14					6	−1.6031E−51		
15					7	3.0784E−68		
16					8	−1.5846E−87		
17		Steps: 1) Enter numbers 1 though 30 in cells E9 to E38 2) Enter formula in cell F9, then copy it into cells F10 to F38. 3) Enter formula in cell C11 4) Enter any Fourier Number in cell C8 and radial location/(characteristic dimension) in cell C9 For a sphere characteristic dimension is radius 5) If temperature ratio is desired at the center of a sphere, do not use r = 0, instead use a very small number in cell C9, e.g. 0.00001 6) The result is shown in cell C11			9	2.186E−109		
18					10	−8.086E−134		
19					11	8.015E−161		
20					12	−2.13E−190		
21					13	1.517E−222		
22					14	−2.896E−257		
23					15	1.482E−294		
24					16	0		
25					17	0		
26					18	0		
27					19	0		
28					20	0		
29					21	0		
30					22	0		
31					23	0		
32					24	0		
33					25	0		
34					26	0		
35					27	0		
36					28	0		
37					29	0		
38					30	0		

■ **Figure E4.22** Spreadsheet solution (sphere) for Example 4.26.

	A	B	C	D	E	F	G	H	I	J	K	L
1												
2		INFINITE CYLINDER - Negligible Surface Resistance to Heat Transfer										
3												
4												
5		Domain: 0 < r < d_c (where d_c is the characteristic dimension, or radius of infinite cylinder)										
6												
7							Terms of Series	J0 for -3 < x < 3			J1 for 3 < x < inf	
8		Fourier Number	0.2		n	lambda_n	ArgJ0	J0			J1(lambda_n)	term_n
9		r/d_c	0		0	2.4048255577	0	1	0.820682791	0.195379932	0.519147809	0.251944131
10					1	5.5200781103	0	1	0.802645916	3.23090329	−0.340264805	−0.00120098.5
11		Temperature Ratio	0.5015		2	8.6537279129	0	1	0.799856138	6.340621261	0.271452299	1.33199E−07
12					3	11.7915344391	0	1	0.79895277	9.467043934	−0.232459829	−3.0563E−13
13					4	14.9309177086	0	1	0.798552578	12.5997901	0.206546432	1.4036E−20
14					5	18.0710639679	0	1	0.798341245	15.73559327	−0.187728803	−1.27219E−29
15					6	21.2116366299	0	1	0.798216311	18.87310399	0.173265895	2.25935E−40
16					7	24.3524715308	0	1	0.798136399	22.01166454	−0.161701553	−7.82877E−53
17					8	27.4934791320	0	1	0.798082224	25.15091632	0.152181217	5.27858E−67
18					9	30.6346064684	0	1	0.798043816	28.29064728	−0.144165981	−6.91297E−83
19												
20												
21												
22												
23												
24												
25												
26												
27												
28												
29												

Steps:

1) Enter numbers 0 to 9 in cells E9 to E18
2) Enter coefficients as indicated in cells F9 to F18
3) Enter formulas for cells G9, H9, I9, J9, K9 and L9, and copy into cells G10 to G18, H10 to H18, I10 to I18, J10 to J18, K10 to K18 and L10 to L18, respectively.
4) Enter formula for cell C11
5) Enter any Fourier Number in cell C8, and r/(characteristic dimension) in cell C9
6) Results are shown in cell C11

Formulas:
Cell C11 = 2*SUM(L9:L18)
Cell G9 = $C$9*F9
Cell H9 = 1−2.2499997*(G9/3)^2+1.2656208*(G9/3)^4−0.3163866*(G9/3)^6 + 0.0444479 *(G9/3)^8 − 0.0039444*(G9/3)^10 + 0.00021*(G9/3)^12
Cell I9 = 0.79788456 + 0.00000156*(3/F9) + 0.01659667*(3/F9)^2 + 0.00017105*(3/F9)^3− 0.00249511*(3/F9)^4 + 0.00113653*(3/F9)^5 − 0.00020033*(3/F9)^6
Cell J9 = F9−2.35619449 + 0.12499612*(3/F9) + 0.0000565*(3/F9)^2 − 0.00637879*(3/F9)^3 + 0.00074348*(3/F9)^4 + 0.00079824*(3/F9)^5 − 0.00029166*(3/F9)^6
Cell K9 = F9^(−1/2)*I9*COS(J9)
Cell L9 = EXP(−F9*F9*$C$8)*H9/(F9*K9)

Figure E4.23 Spreadsheet solution (infinite cylinder) for Example 4.26.

	A	B	C	D	E	F	G	H	I	J
1										
2		INFINITE SLAB - Negligible Surface Resistance to Heat Transfer								
3										
4										
5		Domain: −d_c<x<d_c where d_c is the characteristic dimension, or half-thickness of a slab								
6						=(2*E9+1)/2*PI()				
7							Terms of Series			
8		Fo	1		n	lambda_n	term_n	=(2*(−1)^E9)*EXP(−F9*F9*$C$8)/F9*COS(F9*$C$9)		
9		x/d_c	0		0	1.570796327	0.107977045			
10					1	4.71238898	−9.62899−11			
11		Temperature Ratio	0.108		2	7.853981634	4.13498E−28			
12					3	10.99557429	−5.65531E−54			
13		=+SUM(G9:G39)			4	14.13716694	2.25317E-88			
14					5	17.27875959	−2.5264E−131			
15					6	20.42035225	7.8374E−183			
16					7	23.5619449	−6.6622E−243			
17					8	26.70353756	0			
18					9	29.84513021	0			
19					10	32.98672286	0			
20					11	36.12831552	0			
21					12	39.26990817	0			
22					13	42.41150082	0			
23					14	45.55309348	0			
24					15	48.69468613	0			
25					16	51.83627878	0			
26					17	54.97787144	0			
27					18	58.11946409	0			
28					19	61.26105675	0			
29					20	64.4026494	0			
30					21	67.54424205	0			
31					22	70.68583471	0			
32					23	73.82742736	0			
33					24	76.96902001	0			
34					25	80.11061267	0			
35					26	83.25220532	0			
36					27	86.39379797	0			
37					28	89.53539063	0			
38					29	92.67698328	0			
39					30	95.81857593	0			

Steps:

1) Enter numbers 0 to 30 in cells E9 to E39
2) Enter formula in cell F9
3) Copy formula from cell F9 to cells F10 to F39
4) Enter formula in cell G9
5) Copy formula from cell G9 to cells G10 to G39
6) Enter formula in cell C11
7) Result is shown in cell C11
8) Enter any Fourier number in cell C8 and x/d_c in cell C9 and the results will be shown in cell C11

Figure E4.24 Spreadsheet solution (infinite slab) for Example 4.26.

These spreadsheets also include results for temperature ratios for arbitrarily selected Fourier numbers. The calculated results for temperature ratios compare favorably with those estimated from Figures 4.35, 4.36, and 4.37.

4.5.7 Use of f_h and j Factors in Predicting Temperature in Transient Heat Transfer

In many problems common to food processing, an unknown temperature is determined after the unaccomplished temperature fraction has decreased to less than 0.7. In such cases, the series solutions of the governing partial differential equation (Eq. (4.149)) is simplified. Since only the first term of the series is significant, all remaining terms become small and negligible. This was recognized by Ball (1923), who developed a mathematical approach to predicting temperatures in foods for calculations of thermal processes. We will consider Ball's method of thermal process calculations in more detail in Chapter 5. However, the approach to predict temperature for longer time durations is presented here.

In Equation (4.160) we noted that the unaccomplished temperature fraction decays exponentially. Thus, for a general case we may write

$$\frac{T_a - T}{T_a - T_i} = a_1\, e^{-b_1 t} + a_2\, e^{-b_2 t} + a_3\, e^{-b_3 t \ldots} \tag{4.170}$$

For long time durations, only the first term in the series is significant. Therefore,

$$\frac{T_a - T}{T_a - T_i} = a_1\, e^{-b_1 t} \tag{4.171}$$

or, rearranging,

$$\ln\left[\frac{(T_a - T)}{a_1(T_a - T_i)}\right] = -\, b_1 t \tag{4.172}$$

Ball used two factors to describe heat transfer equation, a time factor called f_h and a temperature lag factor called j_c. To be consistent with Ball's method, we will replace symbol a_1 with j_c, and b_1 with 2.303/f_h. The factor 2.303 is due to the conversion of log scale from base 10 to base e. Substituting these symbols in Equation (4.172),

$$\ln\frac{(T_a - T)}{j_c(T_a - T_i)} = -\frac{2.303}{f_h}t \tag{4.173}$$

Rearranging,

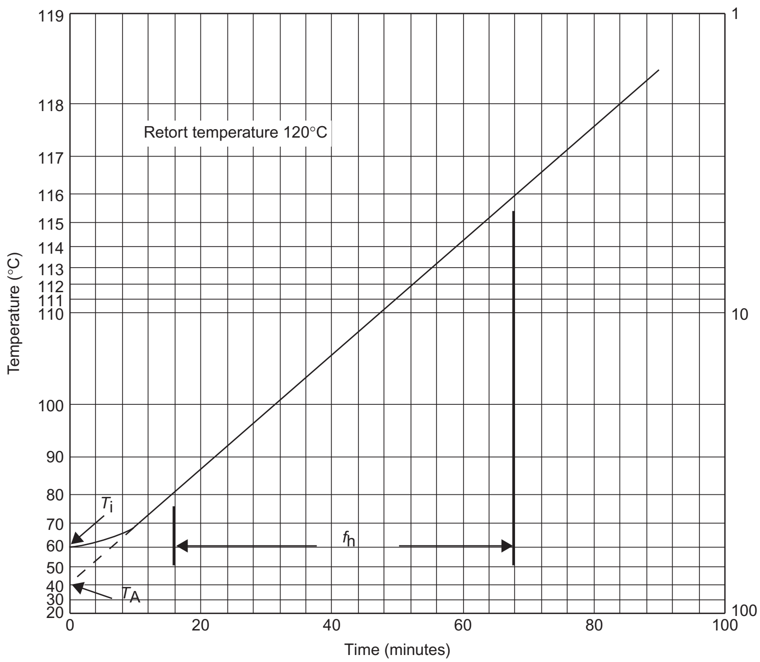
$$\ln(T_a - T) = -\frac{2.303t}{f_h} + \ln\left[j_c(T_a - T_i)\right] \tag{4.174}$$

Converting to log10,

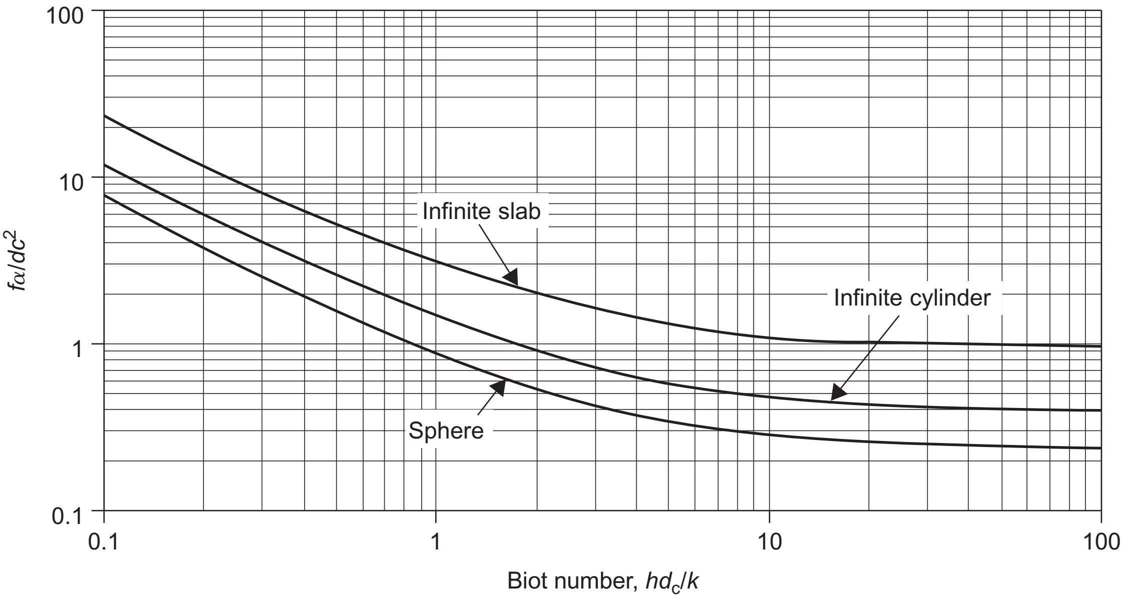
$$\log(T_a - T) = -\frac{t}{f_h} + \log\left[j_c(T_a - T_i)\right] \tag{4.175}$$

Ball used Equation (4.175) in developing his mathematical approach. He plotted the unaccomplished temperature ($T_a - T$) against time t on a log-linear graph rotated by 180°, as shown in Figure 4.39. From the plot, he obtained f_h as the time required for the straight line portion to traverse one log cycle. In other words, f_h is the time taken for the unaccomplished temperature to decrease by 90%. The j_c factor was obtained by extending the straight line to time 0 to obtain an intercept with the ordinate as $T_a - T_A$. Then j_c was defined as $(T_a - T_A)/(T_a - T_i)$. A similar procedure, used for a cooling curve, will be illustrated with an example at the end of this section.

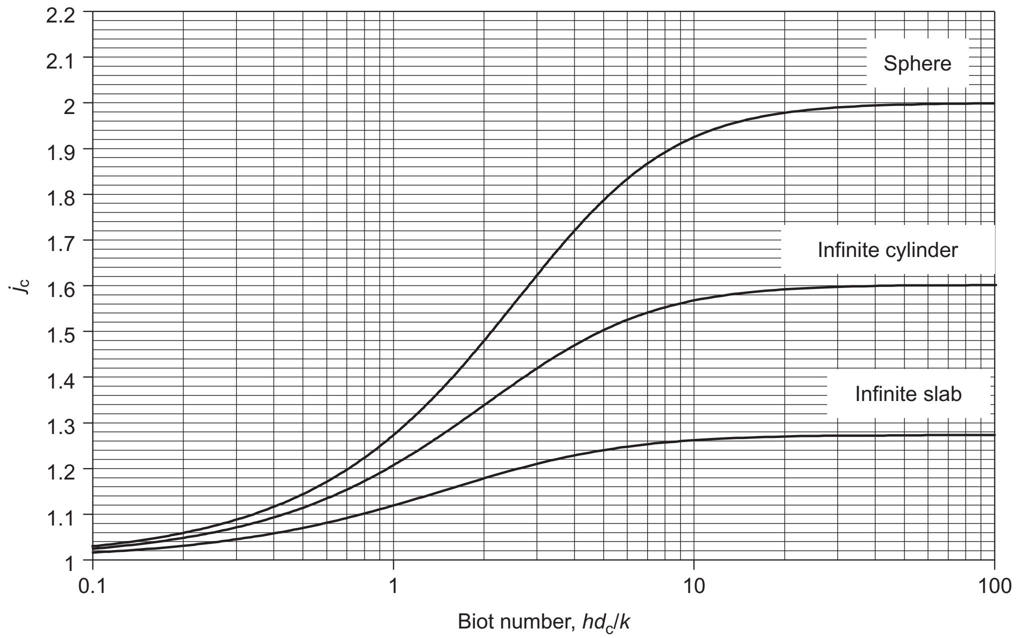
The exact solutions of the first term of infinite series shown in Equations (4.161), (4.163), and (4.165) can either be programmed into a spreadsheet or plotted as originally done by Pflug et al. (1965) as f_h vs N_{Bi}, j_c vs N_{Bi}, and j_m vs N_{Bi}, shown in Figures 4.40, 4.41, and 4.42. Factor j_c is for the temperature lag at the center of an object, and j_m is the mean temperature lag for the object. The f_h value is the same for either center temperature or mean temperature. It is useful to know the mean temperature of an object when determining the



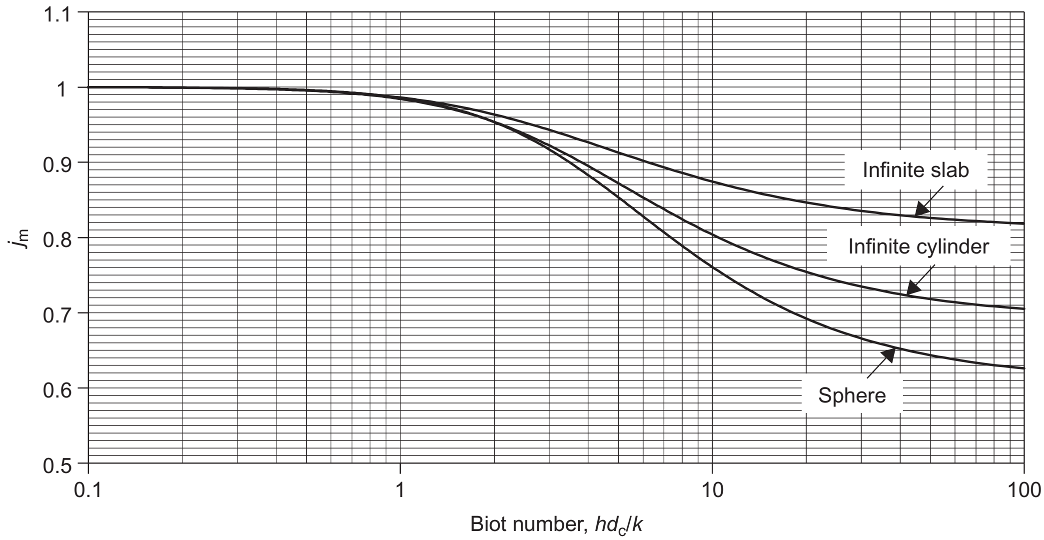
■ **Figure 4.39** Heating curve plotted on a semi-log paper rotated 180°.



■ **Figure 4.40** Heating rate parameter, f_h, as a function of Biot number.



■ **Figure 4.41** Lag factor, j_c, at the geometric center of a sphere, infinite cylinder, and infinite slab as a function of Biot number.



W

Figure 4.42 Average lag factor, j_m of a sphere, infinite cylinder, and infinite slab as a function of Biot number.

heat load required for heating/cooling applications. Using Figures 4.40, 4.41, or 4.42, the f_h, j_c, or j_m factors are obtained for simplified geometrical shapes and used in Equation (4.174) to calculate the temperature at any time. If the shape of the object is a finite cylinder, then we can obtain the f_h and j_c factors using the following relations:

$$\frac{1}{f_{\text{finite cylinder}}} = \frac{1}{f_{\text{infinite cylinder}}} + \frac{1}{f_{\text{infinite slab}}} \tag{4.176}$$

and

$$j_{\text{c,finite cylinder}} = j_{\text{c,infinite cylinder}} \times j_{\text{c,infinite slab}} \tag{4.177}$$

For a brick-shaped object,

$$\frac{1}{f_{\text{brick}}} = \frac{1}{f_{\text{infinite, slab}_1}} + \frac{1}{f_{\text{infinite, slab}_2}} + \frac{1}{f_{\text{infinite, slab}_3}} \tag{4.178}$$

and

$$j_{\text{c,brick}} = j_{\text{c,infinite slab}_1} \times j_{\text{c,infinite slab}_2} \times j_{\text{c,infinite slab}_3} \tag{4.179}$$

The usefulness of the product rule is limited for shapes with dimension ratios much greater than 1 (Pham, 2001). A gross overestimate of cooling time may occur. In those cases, f_h and j_c for finite shapes may be calculated using empirical methods given by Lin et al. (1996).

Example 4.29

A solid food in a cylindrical container is cooled in a 4°C immersion water cooler. Estimate the *f* and *j* factors if the following data were obtained for temperature at the geometric center of the container.

Time (minutes)	Temperature (°C)
0	58
5	48
10	40
15	26
20	25
25	19
30	15
35	12
40	10
45	9
50	7.5
55	7
60	6.5

Given

Cooling medium temperature = 4°C

Approach

We will use a two-cycle log paper and create a scale for the y-axis as shown in Figure E4.25. The straight line portion will be extended to intersect the y-axis to determine the pseudo initial temperature. The f_c factor will be obtained by determining the time traversed for one log cycle change in temperature.

Solution

1. *As shown in Figure E4.25 the y axis is labeled starting from the bottom with 1 + 4°C = 5°C scale; label the rest of the numbers on the y-axis according to the log scale.*
2. *Locate all the points for temperature and time on the graph. Extend the straight line to intersect the y-axis to determine the pseudo initial temperature. From the plot, this temperature is 69°C.*

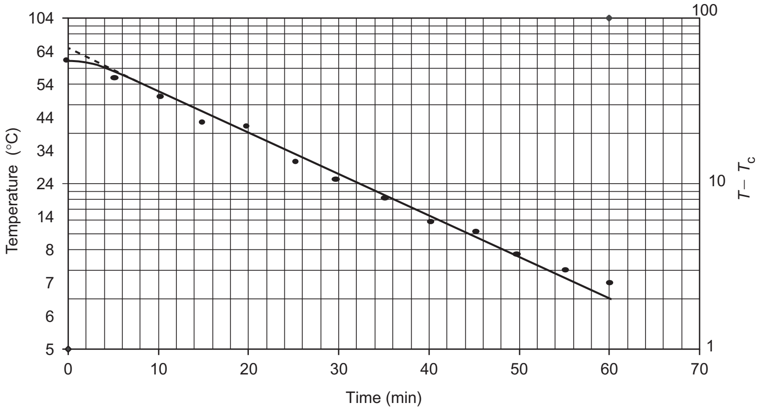


Figure E4.25 Cooling curve for data given in Example 4.27.

3. *Determine the time for the straight line to traverse one log cycle to obtain f_c. From the plot we get $f_c = 40$ min.*
4. *The j_c value is then obtained as*

$$j_c = \frac{69 - 4}{58 - 4} = 1.2$$

5. *For the given data, the f_c is 40 min and j_c is 1.2.*

Example 4.30

A hot dog is initially at 5°C and it is being heated in hot water at 95°C. The convective heat transfer coefficient is 300° W/(m^2 °C). The dimensions of the hot dog are 2 cm diameter and 15 cm long. Assuming that the heat transfer is largely in the radial direction, estimate the product temperature at the geometric center after 10 minutes. The properties of the hot dog are as follows: density = 1100 kg/m^3, specific heat = 3.4 kJ/(kg °C), and thermal conductivity = 0.48 W/(m °C).

Given

Initial temperature = 5°C
Heating medium temperature = 95°C
Heat transfer coefficient = 300 W/(m^2 °C)
Hot dog length = 15 cm
Hot dog diameter = 2 cm
Heating time = 10 min

Density = 1100 kg/m^3
Specific heat = 3.4 kJ/(kg °C)
Thermal conductivity = 0.48 W/(m °C)

Approach

We will consider Pflug's charts for solving this problem. First we will calculate the Biot number and use Figures 4.40 and 4.41 to obtain f_h and j_c factors. The required temperature will be obtained from Equation (4.176).

Solution

1. *Biot number for an infinite cylinder is obtained as*

$$N_{Bi} = \frac{300\ [W/(m^2\ °C)] \times 0.01[m]}{0.48\ [W/(m\ °C)]}$$

$$N_{Bi} = 6.25$$

2. *From Figure 4.40, we obtain for an infinite cylinder,*

$$\frac{f_h \alpha}{d_c^2} = 0.52$$

$$f_h = \frac{0.52 \times (0.01)^2\ [m^2] \times 1100\ [kg/m^3] \times 3400\ [J/(kg\ °C)]}{0.48[W/(m\ °C)]}$$

$$f_h = 405.17\ s$$

3. *From Figure 4.41, we obtain for an infinite cylinder,*

$$j_c = 1.53$$

4. *Using Equation (4.176),*

$$\log(95 - T) = -\frac{10 \times 60\ [s]}{405.17\ [s]} + \log(1.53[95 - 5])$$

$$T = 90.45°C$$

5. *The temperature at the geometric center after 10 min of heating is 90.45°C. The validity of the method may be checked by calculating the unaccomplished temperature fraction at 10 min. For this example, the method is valid because the fraction is $0.05 \ll 0.7$, the assumption used in this method.*

4.6 ELECTRICAL CONDUCTIVITY OF FOODS

It is well known that when electrolytes are placed in an electric field, the ions present within the electrolyte move toward the electrodes with opposite charge. The movement of ions in the electrolyte generates heat. Similarly, when a food containing ions is placed between two electrodes and alternating current or any other wave form current is passed through the food, the food heats by internal heat generation.

Electrical conductance, κ_E, not to be confused with electrical conductivity, is the reciprocal of electrical resistance, or

$$\kappa_E = \frac{1}{R_E} \tag{4.180}$$

where R_E is the electrical resistance of the food material (ohms).

From Ohm's law, we know that

$$R_E = \frac{E_V}{I} \tag{4.181}$$

where E_V is the applied voltage (V), and I is the electric current (A).

Then, electrical conductance is

$$\kappa_E = \frac{1}{E_V} \tag{4.182}$$

Electrical conductivity, σ_E, is a measure of a material's ability to conduct electric current. It is equal to electrical conductance measured between the opposite faces of a 1-meter cube of the material.

$$\sigma_E = \frac{\kappa_E L}{A} = \frac{IL}{E_V A} \tag{4.183}$$

where A is the area (m^2) and L is the length (m). The SI units of electrical conductivity are siemens/m or S/m.

Electrical conductivity of a food material is measured using an electrical conductivity cell, as shown in Figure 4.43. In this cell, a food sample is placed between two electrodes and the electrodes are connected to a power supply. Care is taken to ensure that the electrodes make a firm contact with the food sample.

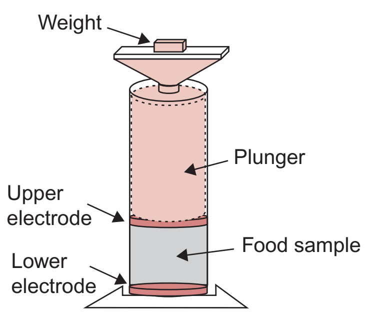


Figure 4.43 A setup for measurement of electrical conductivity of foods. (Adapted from Mitchell and Alvis, 1989)

Electrical conductivity of foods increases with temperature in a linear manner. The following equation may be used to calculate electrical conductivity of a food:

$$\sigma_E = \sigma_o(1 + m''T) \tag{4.184}$$

where σ_o is the electrical conductivity at 0°C (S/m), m'' is coefficient, (1/°C); T is temperature, (°C).

If a reference temperature other than 0°C is chosen then an alternate expression for estimating electrical conductivity is as follows:

$$\sigma_E = \sigma_{ref}[1 + K(T - T_{ref})] \tag{4.185}$$

Values of σ_o, σ_{ref} and coefficients m'' and K for different foods are given in Table 4.6.

The electrical conductivity of a food is a function of its composition—the quantity and type of various components present in the food. Foods containing electrolytes such as salts, acids, certain gums, and thickeners contain charged groups that have a notable effect on the value of electrical conductivity. Based on experimental studies, researchers have reported mathematical relationships to predict

Table 4.6 Coefficients for Equations (4.184) and (4.185) to Estimate Electrical Conductivity

Product	σ_{25} (S/m)	K (°C^{-1})	σ_o (S/m)	m'' (°C^{-1})
Potato	0.32	0.035	0.04	0.28
Carrot	0.13	0.107	− 0.218	− 0.064
Yam	0.11	0.094	− 0.149	− 0.07
Chicken	0.37	0.019	0.194	0.036
Beef	0.44	0.016	0.264	0.027
Sodium Phosphate 0.025 M	0.189	0.027	0.614	0.083
Sodium Phosphate 0.05 M	0.361	0.022	0.162	0.048
Sodium Phosphate 0.1 M	0.676	0.021	0.321	0.0442

Source: Palaniappan and Sastry (1991)

electrical conductivity based on selected food constituents. In fruit juices, the inert suspended solids in the form of pulp or other cellular material act as insulators and tend to decrease the electrical conductivity of the liquid media. Sastry and Palaniappan (1991) obtained the following relationship to describe the effect of solid concentration on electrical conductivity of orange and tomato juices:

$$\sigma_{\mathrm{T,tomato}} = 0.863[1 + 0.174(T - 25)] - 0.101 \times M_s \tag{4.186}$$

$$\sigma_{\mathrm{T,orange}} = 0.567[1 + 0.242(T - 25)] - 0.036 \times M_s \tag{4.187}$$

where M_s is the solid concentration (percent), and T is temperature (°C).

Example 4.31

Estimate the electrical conductivity of 0.1 *M* Sodium Phosphate solution at 30°C.

Given

Temperature = 30°C
0.1 M Sodium Phosphate solution

Approach

We will use Equation (4.184) to determine the electrical conductivity.

Solution

1. *Using Equation (4.184) and appropriate values for electrical conductivity at the reference temperature of 0°C and coefficient m″, we obtain*

$$\sigma_E = 0.321(1 + 0.0442 \times 30)$$

$$\sigma_E = 0.746 \text{ S/m}$$

2. *Note that if we use Equation (4.185) with appropriate values of electrical conductivity at reference temperature of 25°C and coefficient K, we obtain*

$$\sigma_E = 0.676(1 + 0.021(30 - 25))$$

$$\sigma_E = 0.747 \text{ S/m}$$

As expected, we get the same result using Equation (4.184) or (4.185).

4.7 OHMIC HEATING

In ohmic heating, main alternating current is passed directly through a conductive food, which causes heat generation within the food. Due to internal heat generation, the heating is rapid and more uniform than traditional systems used for heating foods where heat must travel from the outside surface to the inside of the food. The rapid and uniform heating of a food is advantageous in retaining many quality characteristics such as color, flavor, and texture. The efficiency of ohmic heating is dependent upon how well the electric current can pass through the food, as determined by its electrical conductivity. Therefore, the knowledge of electrical conductivity of foods is important in designing processes and equipment involving ohmic heating.

As an example of ohmic heating, we will consider heating a liquid food with Newtonian characteristics when pumped through an ohmic heater. We assume that the flow conditions through the tubular-shaped heater are similar to plug flow, and a constant voltage gradient exists along the heater. In this setup, heat is generated within the liquid due to ohmic heating, and heat loss from the fluid is in radial direction to the outside, if the heater pipe is uninsulated.

For this setup, conducting a heat balance we get,

$$\dot{m}c_p\frac{\mathrm{d}T}{\mathrm{d}t} = (|\Delta V|^2\sigma_o(1+m''T))\left(\frac{\pi d_c^2 L}{4}\right) - U\pi d_c L(T-T_\infty) \qquad (4.188)$$

where $|\Delta V|$ is voltage gradient along the heater pipe length, (V/m); σ_o is the electrical conductivity at 0°C; m is the slope obtained from Equation (4.184); d_c is the characteristic dimension or diameter of the heater pipe (m); L is the length of heater pipe (m); U is the overall heat transfer coefficient based on the inside area of the heater pipe, (W/m^2 °C); T_∞ is the temperature of the air surrounding the heater (°C).

The initial condition is

$$t = 0, \quad T = T_o$$

Expanding the terms in Equation (4.188) and rearranging, we get

$$\frac{\mathrm{d}T}{\mathrm{d}t} = \frac{\alpha\pi d_c LT}{\dot{m}c_p} + \frac{b\pi DL}{\dot{m}c_p} \qquad (4.189)$$

where

$$a = \frac{|\Delta V|^2 \, d_c \sigma_o m''}{4} - U \tag{4.190}$$

$$b = \frac{d_c |\Delta V|^2 \, \sigma_o}{4} + UT_\infty \tag{4.191}$$

Integrating Equation (4.189), we obtain

$$\frac{aT + b}{aT_o + b} = e^{\left(\frac{a\pi d_c L}{\dot{m} c_p}\right)} \tag{4.192}$$

In the following example, we will use Equation (4.192) to determine the temperature of a liquid exiting an ohmic heater.

Example 4.32

A liquid food is being pumped through an ohmic heater at 0.5 kg/s. The inside diameter of the heater pipe is 0.05 m and it is 3 m long. The specific heat of the liquid food is 4000 J/kg °C. The applied voltage is 15,000 V. The overall heat transfer coefficient based on the inside pipe area is 100 W/m^2 °C. The surrounding temperature of the air is 20°C. The liquid food enters the ohmic heater at 50°C. Assume that the properties of the liquid food are similar to 0.05 M Sodium Phosphate solution. Calculate the temperature at which the liquid food exits.

Given

Flow rate = 0.5 kg/s
Inside diameter of ohmic heater = 0.05 m
Length of ohmic heater = 3 m
Specific heat capacity = 4000 J/kg °C
Voltage = 15000 V
Overall heat transfer coefficient = 100 W/m^2 °C
Surrounding air temperature = 20°C
Inlet temperature = 50°C

Approach

Using the given information, we will determine the voltage gradient, and obtain electrical properties using Table 4.6 for 0.05 M Sodium Phosphate. Using Equation (4.192) we will calculate the liquid temperature at the exit of the heater.

Solution

1. *Voltage gradient is obtained from the given information as*

$$|\Delta V| = \frac{15000}{3} = 5000\ V/m$$

2. *From Table 4.6, for 0.05 M Sodium Phosphate,*

$$\sigma_o = 0.162\ S/m$$

$$m'' = 0.048(°C^{-1})$$

3. *Using Equations (4.190) and (4.191), we obtain a and b values as follows:*

$$a = 2330\ W/m^2\ °C$$

$$b = 52625\ W/m^2\ °C$$

4. *Substituting a and b values in Equation (4.192)*

$$\frac{2330T + 52625}{2330 \times 50 + 52625} = e^{\left(\frac{\pi \times 2330 \times 0.05 \times 3}{0.5 \times 4000}\right)}$$

Solving for the unknown temperature, T, we get

$$T = 103°C$$

The temperature of the liquid food will increase from 50 to 103°C when heated in the ohmic heater.

4.8 MICROWAVE HEATING

Electromagnetic radiation is classified by wavelength or frequency. The electromagnetic spectrum between frequencies of 300 MHz and 300 GHz is represented by microwaves. Since microwaves are used in radar, navigational equipment, and communication equipment, their use is regulated by governmental agencies. In the United States, the Federal Communications Commission (FCC) has set aside two frequencies for industrial, scientific, and medical (ISM) apparatus in the microwave range, namely 915 ± 13 MHz, and 2450 ± 50 MHz. Similar frequencies are regulated worldwide through the International Telecommunication Union (ITU).

Microwaves have certain similarities to visible light. Microwaves can be focused into beams. They can transmit through hollow tubes. Depending on the dielectric properties of a material, they may be reflected or absorbed by the material. Microwaves may also transmit through materials without any absorption. Packaging materials such as glass, ceramics, and most thermoplastic materials allow microwaves to pass through with little or no absorption. When traveling from one material to another, microwaves may change direction, similar to the bending of light rays when they pass from air to water.

In contrast to conventional heating systems, microwaves penetrate a food, and heating extends within the entire food material. The rate of heating is therefore more rapid. Note that microwaves generate heat due to their interactions with the food materials. The microwave radiation itself is nonionizing radiation, distinctly different from ionizing radiation such as X-rays and gamma rays. When foods are exposed to microwave radiation, no known nonthermal effects are produced in food material (IFT, 1989; Mertens and Knorr, 1992).

The wavelength, frequency, and velocity of electromagnetic waves are related by the following expression

$$\lambda = u/f' \tag{4.193}$$

where, λ is wavelength in meters; f' is frequency in hertz; u is the speed of light (3×10^8 m/s).

Using Equation (4.193), the wavelengths of the permitted ISM frequencies in the microwave range can be calculated as

$$\lambda_{915} = \frac{3 \times 10^8 (m/s)}{915 \times 10^6 (1/s)} = 0.328 \text{ m}$$

and

$$\lambda_{2450} = \frac{3 \times 10^8 (m/s)}{2450 \times 10^6 (1/s)} = 0.122 \text{ m}$$

4.8.1 Mechanisms of Microwave Heating

The absorption of microwaves by a dielectric material results in the microwaves giving up their energy to the material, with a consequential rise in temperature. The two important mechanisms that explain heat generation in a material placed in a microwave field are ionic polarization and dipole rotation.

4.8.1.1 *Ionic Polarization*

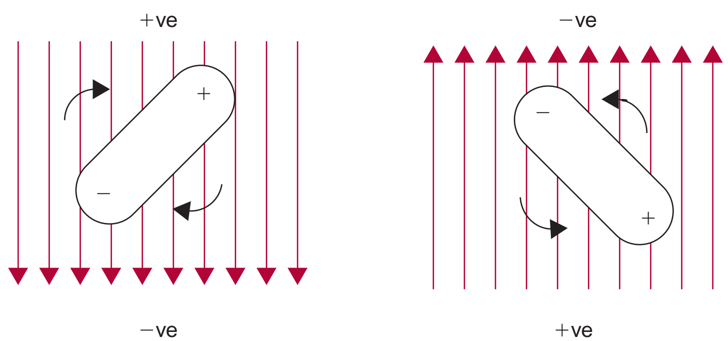
When an electrical field is applied to food solutions containing ions, the ions move at an accelerated pace due to their inherent charge. The resulting collisions between the ions cause the conversion of kinetic energy of the moving ions into thermal energy. A solution with a high concentration of ions would have more frequent ionic collisions and therefore exhibit an increase in temperature.

4.8.1.2 *Dipole Rotation*

Food materials contain polar molecules such as water. These molecules generally have a random orientation. However, when an electrical field is applied, the molecules orient themselves according to the polarity of the field. In a microwave field, the polarity alternates rapidly (e.g., at the microwave frequency of 2450 MHz, the polarity changes at 2.45×10^9 cycles per second). The polar molecules rotate to maintain alignment with the rapidly changing polarity (Fig. 4.44). Such rotation of molecules leads to friction with the surrounding medium, and heat is generated. With increasing temperatures, the molecules try to align more rapidly with the applied field. Several factors influence the microwave heating of a material, including the size, shape, state (e.g., water or ice), and properties of the material, and the processing equipment.

4.8.2 Dielectric Properties

In microwave processing, we are concerned with the electrical properties of the material being heated. The important electrical properties are the relative dielectric constant ε' and the relative dielectric loss ε''.



W

Figure 4.44 Movement of a dipole in an electrical field. (From Decareau and Peterson, 1986)

The term loss implies the conversion (or "loss") of electrical energy into heat, and the term *relative* means relative to free space.

The relative dielectric constant ε' expresses the ability of the material to store electrical energy, and the relative dielectric loss ε'' denotes the ability of the material to dissipate the electrical energy. These properties provide an indication of the electrical insulating ability of the material. Foods are in fact very poor insulators; therefore, they generally absorb a large fraction of the energy when placed in a microwave field, resulting in instantaneous heating (Mudgett, 1986). The dielectric loss factor for the material, ε'', which expresses the degree to which an externally applied electrical field will be converted to heat, is given by

$$\varepsilon'' = \varepsilon' \tan \delta \tag{4.194}$$

The loss tangent, $\tan \delta$, provides an indication of how well the material can be penetrated by an electrical field and how it dissipates electrical energy as heat.

4.8.3 Conversion of Microwave Energy into Heat

Microwave energy in itself is not thermal energy; rather, heating is a consequence of the interactions between microwave energy and a dielectric material. The conversion of the microwave energy to heat can be approximated with the following equation (Copson, 1975; Decareau and Peterson, 1986):

$$P_D = 55.61 \times 10^{-14} E^2 f' \varepsilon' \tan \delta \tag{4.195}$$

where P_D is the power dissipation (W/cm^3); E is electrical field strength (V/cm); f' is frequency (Hz); ε' is the relative dielectric constant, and $\tan \delta$ is the loss tangent.

In Equation (4.195), the dielectric constant ε' and the loss tangent $\tan \delta$ are the properties of the material, and the electrical field strength E and frequency f' represent the energy source. Thus, there is a direct relationship between the material being heated and the microwave system providing the energy for heating. It is evident in Equation (4.195) that increasing the electrical field strength has a dramatic effect on the power density, since the relationship involves a square term.

The governing heat transfer equation presented earlier in this chapter, Equation (4.149), can be modified for use in predicting heat transfer in a material placed in a microwave field. A heat generation term q''' equivalent to the power dissipation obtained from Equation (4.195) is introduced in Equation (4.149). Thus, for transient heat transfer in an infinite slab, we can obtain the following expression for a one-dimensional case:

$$\frac{\partial^2 T}{\partial x^2} + \frac{q'''}{k} = \frac{\rho c_p \partial T}{k \partial t} \tag{4.196}$$

Numerical techniques are used to solve the preceding equation (Mudgett, 1986).

4.8.4 Penetration Depth of Microwaves

The energy transfer between microwaves and the material exposed to the microwave field is influenced by the electrical properties of the material. The distribution of energy within a material is determined by the attenuation factor α'.

The attenuation factor α' is calculated from the values for the loss tangent, relative dielectric constant, and the frequency of the microwave field:

$$\alpha' = \frac{2\pi}{\lambda}\left[\frac{\varepsilon'}{2}\left(\sqrt{1+\tan^2\delta} - 1\right)\right]^{1/2} \tag{4.197}$$

The penetration of an electrical field can be calculated from the attenuation factor. As shown by von Hippel (1954), the depth Z below the surface of the material at which the electrical field strength is 1/e that of the electrical field in the free space, is the inverse of the attenuation factor. Thus,

$$Z = \frac{\lambda}{2\pi}\left[\frac{2}{\varepsilon'\left(\sqrt{1+\tan^2\delta} - 1\right)}\right]^{1/2} \tag{4.198}$$

Noting that frequency and wavelength are inversely related, it is evident from Equation (4.198) that microwave energy at 915 MHz penetrates more deeply than at 2450 MHz.

In addition to the foregoing description of penetration of a microwave field in a material, the depth of penetration for microwave power is usually described in two different ways. First, the penetration depth is the distance from the surface of a dielectric material where the incident power is decreased to 1/e of the incident power. Lambert's expression for power absorption gives

$$P = P_0 \, e^{-2\alpha' d} \tag{4.199}$$

where P_0 is the incident power, P is the power at the penetration depth, d is the penetration depth, and α' is the attenuation factor.

If the power is reduced to 1/e of the incident power at depth d, we have $P/P_0 = 1/e$. Therefore, from Equation (4.199), $2\alpha' d = 1$ and $d = 1/2\alpha'$.

The second definition of the penetration depth is stated in terms of half-power depth (i.e., one-half of the incident power). Therefore, at half-power depth, $P/P_0 = 1/2$. From Equation (4.199), $e^{-2}\alpha' d = 1/2$, and solving for d we get $d = 0.347/\alpha'$.

Example 4.33

In a paper on microwave properties, Mudgett (1986) provides data on dielectric constants and loss tangents for raw potatoes. For a microwave frequency of 2450 MHz and at 20°C, the dielectric constant is 64 and the loss tangent is 0.23. Determine the attenuation factor, the field penetration depth, and the depth below the surface of a potato at which the microwave power is reduced to one-half of the incident power.

Approach

We will use Equations (4.197) and (4.198) to determine the attenuation factor and the penetration depth for the microwave field, respectively. The distance from the surface of the material at which the power is reduced to one-half of the incident power will be calculated using modifications of Equation (4.199).

Solution

1. *From Equation (4.197):*

$$\alpha' = \frac{2\pi \times 2450 \times 10^6 (1/s)}{3 \times 10^8 (m/s) \times 100 (cm/m)} \left[\frac{64}{2} \left(\sqrt{1 + (0.23)^2} - 1 \right) \right]^{1/2}$$

$$\alpha' = 0.469 \; (cm^{-1})$$

2. *The penetration depth for the microwave field is the inverse of α' as seen in Equation (4.198). Therefore,*

$$\text{Field penetration depth} = Z = 1/\alpha' = 1/0.469 = 2.13 \text{ cm}$$

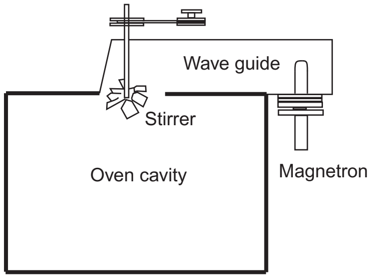
3. *To obtain the half-power depth of penetration, we use the modification of Lambert's expression, Equation (4.199), and solve for d:*

$$d = \frac{0.347}{\alpha'} = \frac{0.347}{0.469} = 0.74 \text{ cm}$$

4. *The half-power depth for potatoes at 2450 MHz and 20°C is calculated to be 0.74 cm.*

4.8.5 Microwave Oven

A typical microwave oven consists of the following major components (Fig. 4.45).



W

Figure 4.45 Major components of a microwave oven.

- Power supply. The purpose of the power supply is to draw electrical power from the line and convert it to the high voltage required by the magnetron. The magnetron usually requires several thousand volts of direct current.
- Magnetron or power tube. The magnetron is an oscillator capable of converting the power supplied into microwave energy. The magnetron emits high-frequency radiant energy. The polarity of the emitted radiation changes between negative and positive at high frequencies (e.g., 2.45×10^9 cycles per second for a magnetron operating at a frequency of 2450 MHz, the most common frequency used for domestic ovens).
- Wave guide or transmission section. The wave guide propagates, radiates, or transfers the generated energy from the magnetron to the oven cavity. In a domestic oven, the wave guide is a few centimeters long, whereas in industrial units it can be a few meters long. The energy loss in the wave guide is usually quite small.
- Stirrer. The stirrer is usually a fan-shaped distributor that rotates and scatters the transmitted energy throughout the oven. The stirrer disturbs the standing wave patterns, thus allowing better energy distribution in the oven cavity. This is particularly important when heating nonhomogenous materials like foods.
- Oven cavity or oven. The oven cavity encloses the food to be heated within the metallic walls. The distributed energy from the stirrer is reflected by the walls and intercepted by the food from many directions with more or less uniform energy density. The energy impinging on the food is absorbed and converted into heat. The size of the oven cavity is influenced by the

wavelength. The length of the cavity wall should be greater than one-half the wavelength and any multiple of a half-wave in the direction of the wave propagation. The wavelength at 2450 MHz frequency was calculated earlier to be 12.2 cm; therefore, the oven cavity wall must be greater than 6.1 cm. The oven cavity door includes safety controls and seals to retain the microwave energy within the oven during the heating process.

4.8.6 Microwave Heating of Foods

Heating of foods in a microwave field offers several advantages over more conventional methods of heating. The following are some of the important features of microwave heating that merit consideration.

4.8.6.1 *Speed of Heating*

The speed of heating of a dielectric material is directly proportional to the power output of the microwave system. In industrial units, the typical power output may range from 5 to 100 kW. Although high speed of heating is attainable in the microwave field, many food applications require good control of the rate at which the foods are heated. Very high-speed heating may not allow desirable physical and biochemical reactions to occur. The speed of heating in a microwave is governed by controlling the power output. The power required for heating is also proportional to the mass of the product.

4.8.6.2 *Frozen Foods*

The heating behavior of frozen foods is markedly influenced by the different dielectric properties of ice and water (Table 4.7). Due to its low dielectric loss factor, ice is more transparent to microwaves than water. Thus, ice does not heat as well as water. Therefore, when using microwaves to temper frozen foods, care is taken to keep the temperature of the frozen food just below the freezing point. If the ice

Table 4.7 Dielectric Properties of Water and Ice at 2450 MHz

	Relative dielectric constant, ε'	Relative dielectric loss constant, ε''	Loss tangent, $\tan\delta$
Ice	3.2	0.0029	0.0009
Water (at 25°C)	78	12.48	0.16

Source: Schiffman (1986)

melts, runaway heating may occur because the water will heat much faster due to the high dielectric loss factor of water.

4.8.6.3 *Shape and Density of the Material*

The shape of the food material is important in obtaining uniformity of heating. Non-uniform shapes result in local heating; similarly, sharp edges and corners cause non-uniform heating.

4.8.6.4 *Food Composition*

The composition of the food material affects how it heats in the microwave field. The moisture content of food directly affects the amount of microwave absorption. A higher amount of water in a food increases the dielectric loss factor ε''. In the case of foods of low moisture content, the influence of specific heat on the heating process is more pronounced than that of the dielectric loss factor. Therefore, due to their low specific heat, foods with low moisture content also heat at acceptable rates in microwaves. If the food material is highly porous with a significant amount of air, then due to low thermal conductivity of air, the material will act as a good insulator and show good heating rates in microwaves.

Another compositional factor that has a marked influence on heating rates in microwaves is the presence of salt. As stated previously, an increased concentration of ions promotes heating in microwaves. Thus, increasing the salt level in foods increases the rate of heating. Although oil has a much lower dielectric loss factor than water, oil has a specific heat less than half that of water. Since a product with high oil content will require much less heat to increase in temperature, the influence of specific heat becomes the overriding factor, and oil exhibits a much higher rate of heating than water (Ohlsson, 1983). More details on these and other issues important during the microwave heating of foods are elaborated by Schiffman (1986) and by Decareau and Peterson (1986).

The industrial applications of microwave processing of foods are mostly for tempering of frozen foods (increasing the temperature of frozen foods to −4 to −20°C), such as meat, fish, butter, and berries; for drying of pasta, instant tea, herbs, mushrooms, fish protein, bread crumbs, onions, rice cakes, seaweed, snack foods, and egg yolk; for cooking of bacon, meat patties, and potatoes; for vacuum drying of citrus juices, grains, and seeds; for freeze-drying of meat, vegetables, and fruits; for pasteurization and sterilization of prepared foods; for baking of bread and doughnuts; and for roasting of nuts, coffee beans, and cocoa beans (Decareau, 1992; Giese, 1992).

PROBLEMS

4.1 Calculate the rate of heat transfer per unit area through a 200-mm-thick concrete wall when the temperatures are 20°C and 5°C on the two surfaces, respectively. The thermal conductivity of concrete is 0.935 W/(m °C).

4.2 A side wall of a storage room is 3 m high, 10 m wide, and 25 cm thick. The thermal conductivity is $k = 0.85$ W/(m °C). If, during the day, the inner surface temperature of the wall is 22°C and the outside surface temperature is 4°C:

a. Using the Thermal Resistance Concept, calculate the resistance to heat transfer for the wall.

b. Calculate the rate of heat transfer through the wall, assuming steady-state conditions.

4.3 An experiment was conducted to measure thermal conductivity of a formulated food. The measurement was made by using a large plane plate of the food material, which was 5 mm thick. It was found, under steady-state conditions, that when a temperature difference of 35°C was maintained between the two surfaces of the plate, a heat-transfer rate per unit area of 4700 W/m^2 was measured near the center of either surface. Calculate the thermal conductivity of the product, and list two assumptions used in obtaining the result.

4.4 Estimate the thermal conductivity of applesauce at 35°C. (Water content = 78.8% wet basis).

4.5 A 20-cm-diameter cooking pan is placed on a stove. The pan is made of steel ($k = 15$ W/[m °C]), and it contains water boiling at 98°C. The bottom plate of the pan is 0.4 cm thick. The inside surface temperature of the bottom plate, in contact with water, is 105°C.

a. If the rate of heat transfer through the bottom plate is 450 W, determine the outside surface temperature of the bottom plate exposed to the heating stove.

b. Determine the convective heat-transfer coefficient for boiling water.

4.6 A 10-m-long pipe has an inside radius of 70 mm, an outside radius of 80 mm, and is made of stainless steel ($k = 15$ W/[m °C]). Its inside surface is held at 150°C, and its outside surface is at 30°C. There is no heat generation, and steady-state conditions hold. Compute the rate at which heat is being transferred across the pipe wall.

4.7 In a multilayered rectangular wall, the thermal resistance of the first layer is 0.005°C/W, the resistance of the second layer is 0.2°C/W, and for the third layer it is 0.1°C/W. The overall temperature gradient in the multilayered wall from one side to another is 70°C.

a. Determine the heat flux through the wall.

b. If the thermal resistance of the second layer is doubled to 0.4°C/W, what will be its influence in % on the heat flux, assuming the temperature gradient remains the same?

4.8 A plain piece of insulation board is used to reduce the heat loss from a hot furnace wall into the room. One surface of the board is at 100°C and the other surface is at 20°C. It is desired to keep the heat loss down to 120 W/m^2 of the insulation board. If the thermal conductivity of the board is 0.05 W/(m °C), calculate the required thickness of the board.

4.9 Consider an ice chest with the following dimensions: length = 50 cm, width = 40 cm, and height = 30 cm, made of a 3-cm-thick insulating material (k = 0.033 W/[m °C]). The chest is filled with 30 kg of ice at 0°C. The inner wall surface temperature of the ice chest is assumed to be constant at 0°C. The latent heat of fusion of ice is 333.2 kJ/kg. The outside wall surface temperature of the chest is assumed to remain constant at 25°C. How long would it take to completely melt the ice? Assume negligible heat transfer through the bottom surface.

***4.10** Steam with 80% quality is being used to heat a 40% total solids tomato purée as it flows through a steam injection heater at a rate of 400 kg/h. The steam is generated at 169.06 kPa and is flowing to the heater at a rate of 50 kg/h. If the specific heat of the product is 3.2 kJ/(kg K), determine the temperature of the product leaving the heater when the initial temperature is 50°C. Determine the total solids content of the product after heating. Assume the specific heat of the heated purée is 3.5 kJ/(kg °C).

---

* Indicates an advanced level of difficulty in solving.

4.11 A stainless steel pipe ($k = 15$ W/[m °C]) with a 2.5-cm inner diameter and 5-cm outer diameter is being used to convey high-pressure steam. The pipe is covered with a 5-cm layer of insulation ($k = 0.18$ W/[m °C]). The inside steel pipe surface temperature is 300°C, and the outside insulation surface temperature is 90°C.

a. Determine the rate of heat transfer per meter length of the pipe.

b. The insulation selected for the purpose has a melting temperature of 220°C. Should you be concerned about the integrity of the insulation for the listed conditions?

4.12 Air at 25°C blows over a heated steel plate with its surface maintained at 200°C. The plate is 50×40 cm and 2.5 cm thick. The convective heat-transfer coefficient at the top surface is 20 W/(m^2 K). The thermal conductivity of steel is 45 W/(m K). Calculate the heat loss per hour from the top surface of the plate.

4.13 A liquid food is being heated in a tubular heat exchanger. The inside pipe wall temperature is 110°C. The internal diameter of the pipe is 30 mm. The product flows at 0.5 kg/s. If the initial temperature of the product is 7°C, compute the convective heat-transfer coefficient. The thermal properties of the product are the following: specific heat = 3.7 kJ/(kg °C), thermal conductivity = 0.6 W/(m °C), product viscosity = 500×10^{-6} Pa s, density = 1000 kg/m^2, product viscosity at 110°C = 410×10^{-6} Pa s.

4.14 Compute the convective heat-transfer coefficient for natural convection from a vertical, 100 mm outside diameter, 0.5 m long, stainless-steel pipe. The surface temperature of the uninsulated pipe is 145°C, and the air temperature is 40°C.

***4.15** A 30 m long pipe with an external diameter of 75 mm is being used to convey steam at a rate of 1000 kg/h. The steam pressure is 198.53 kPa. The steam enters the pipe with a dryness fraction of 0.98 and must leave the other end of the pipe with a minimum dryness fraction of 0.95. Insulation with a thermal conductivity of 0.2 W/(m K) is available. Determine the minimum thickness of insulation necessary.

---

* Indicates an advanced level of difficulty in solving.

The outside surface temperature of insulation is assumed to be 25°C. Neglect the conductive resistance of the pipe material and assume no pressure drop across the pipe.

4.16 Estimate the convective heat-transfer coefficient for natural convection from a horizontal steam pipe. The outside surface temperature of the insulated pipe is 80°C. The surrounding air temperature is 25°C. The outside diameter of the insulated pipe is 10 cm.

4.17 A vertical cylindrical container is being cooled in ambient air at 25°C with no air circulation. If the initial temperature of the container surface is 100°C, compute the surface heat-transfer coefficient due to natural convection during the initial cooling period. The diameter of the container is 1 m, and it is 2 m high.

4.18 Water at a flow rate of 1 kg/s is flowing in a pipe of internal diameter 5 cm. If the inside pipe surface temperature is 90°C and mean bulk water temperature is 50°C, compute the convective heat-transfer coefficient.

4.19 A blower is used to move air through a pipe at a rate of 0.01 kg/s. The inside pipe surface temperature is 40°C. The bulk temperature of the air reduces from 80°C to 60°C as it passes through a 5-m section of the pipe. The inside diameter of the pipe is 2 cm. Estimate the convective heat-transfer coefficient using the appropriate dimensionless correlation.

4.20 Estimate the convective heat-transfer coefficient on the outside of oranges (external diameter = 5 cm) when submerged in a stream of chilled water pumped around the orange. The velocity of water around an orange is 0.1 m/s. The surface temperature of the orange is 20°C and the bulk water temperature is 0°C.

***4.21** A flat wall is exposed to an environmental temperature of 38°C. The wall is covered with a layer of insulation 2.5 cm thick whose thermal conductivity is 1.8 W/(m K), and the temperature of the wall on the inside of the insulation is 320°C. The wall loses heat to the environment by convection. Compute the value of the convection

* Indicates an advanced level of difficulty in solving.

heat-transfer coefficient that must be maintained on the outer surface of the insulation to ensure that the outer surface temperature does not exceed 40°C.

4.22 Steam at 150°C flows inside a pipe that has an inside radius of 50 mm and an outside radius of 55 mm. The convective heat-transfer coefficient between the steam and the inside pipe wall is 2500 W/(m^2 °C). The outside surface of the pipe is exposed to ambient air at 20°C with a convective heat-transfer coefficient of 10 W/(m^2 °C). Assuming steady state and no heat generation, calculate the rate of heat transfer per meter from the steam to the air across the pipe. Assume thermal conductivity of stainless steel is 15 W/(m °C.)

4.23 The outside wall of a refrigerated storage room is 10 m long and 3 m high and is constructed with 100-mm concrete blocks (k = 0.935 W/[m °C]) and 10 cm of fiber insulation board (k = 0.048 W/[m °C]). The inside of the room is at −10°C and the convective heat-transfer coefficient is 40 W/(m^2 K); the outside temperature is 30°C with a convective heat-transfer coefficient of 10 W/(m^2 K) on the outside wall surface. Calculate the overall heat-transfer coefficient.

4.24 In a food processing plant, a steel pipe (thermal conductivity = 17 W/m °C, internal diameter = 5 cm; thickness = 3 mm) is being used to transport a liquid food. The inside surface temperature of the pipe is at 95°C. A 4-cm-thick insulation (thermal conductivity = 0.03 W/m °C) is wrapped around the pipe. The outside surface temperature of the insulation is 30°C. Calculate the rate of heat transfer per unit length of the pipe.

4.25 A walk-in freezer of 4 m width, 6 m length, and 3 m height is being built. The walls and ceiling contain 1.7-mm-thick stainless steel (k = 15 W/[m °C]), 10-cm-thick foam insulation (k = 0.036 W/[m °C]), and some thickness of corkboard (k = 0.043 W/[m °C]) to be established, and 1.27 cm-thickness wood siding (k = 0.104 W/[m °C]). The inside of the freezer is maintained at −40°C. Ambient air outside the freezer is at 32°C. The convective heat-transfer coefficient is 5 W/(m^2 K) on the wood side of the wall and 2 W/(m^2 K) on the steel side. If the outside air has a dew point of 29°C, calculate the thickness of corkboard insulation that would prevent condensation of moisture on the outside wall of the freezer. Calculate the rate of heat transfer through the walls and ceiling of this freezer.

4.26 A liquid food is being conveyed through an uninsulated pipe at 90°C. The product flow rate is 0.25 kg/s and has a density of 1000 kg/m^3, specific heat of 4 kJ/(kg K), viscosity of 8×10^{-6} Pa s, and thermal conductivity of 0.55 W/(m K). Assume the viscosity correction is negligible. The internal pipe diameter is 20 mm with 3 mm thickness made of stainless steel (k = 15 W/[m °C]). The outside temperature is 15°C. If the outside convective heat-transfer coefficient is 18 W/(m^2 K), calculate the steady-state heat loss from the product per unit length of pipe.

***4.27** A liquid food is being pumped in a 1-cm-thick steel pipe. The inside diameter of the pipe is 5 cm. The bulk temperature of liquid food is 90°C. The inside pipe surface temperature is 80°C. The surface heat-transfer coefficient inside the pipe is 15 W/(m^2 K). The pipe has a 2-cm-thick insulation. The outside bulk air temperature is 20°C. The surface heat-transfer coefficient on the outside of insulation is 3 W/(m^2 K).

a. Calculate the insulation surface temperature exposed to the outside.

b. If the pipe length is doubled, how would it influence the insulation surface temperature? Discuss.

4.28 For a metal pipe used to pump tomato paste, the overall heat-transfer coefficient based on internal area is 2 W/(m^2 K). The inside diameter of the pipe is 5 cm. The pipe is 2 cm thick. The thermal conductivity of the metal is 20 W/(m K). Calculate the outer convective heat-transfer coefficient. The inside convective heat-transfer coefficient is 5 W/(m^2 K).

4.29 A cold-storage room is maintained at −18°C. The internal dimensions of the room are 5 m × 5 m × 3 m high. Each wall, ceiling, and floor consists of an inner layer of 2.5 cm thick wood with 7 cm thick insulation and an 11 cm brick layer on the outside. The thermal conductivities of respective materials are wood 0.104 W/(m K), glass fiber 0.04 W/(m K), and brick 0.69 W/(m K). The convective heat-transfer coefficient for wood to still air is 2.5 W/(m^2 K), and from moving air

---

* Indicates an advanced level of difficulty in solving.

to brick is 4 W/(m^2 K). The outside ambient temperature is 25°C. Determine:

a. The overall heat-transfer coefficient.
b. The temperature of the exposed surfaces.
c. The temperatures of the interfaces.

4.30 Steam at 169.60 kPa is condensed inside a pipe (internal diameter = 7 cm, thickness = 3 mm). The inside and outside convective heat-transfer coefficients are 1000 and 10 W/(m^2 K), respectively. The thermal conductivity of the pipe is 45 W/(m K). Assume that all thermal resistances are based on the outside diameter of the pipe, and determine the following:

a. Percentage resistance offered by the pipe, by the steam, and by the outside.
b. The outer surface temperature of the pipe if the temperature of the air surrounding the pipe is 25°C.

4.31 A steel pipe (outside diameter 100 mm) is covered with two layers of insulation. The inside layer, 40 mm thick, has a thermal conductivity of 0.07 W/(m K). The outside layer, 20 mm thick, has a thermal conductivity of 0.15 W/(m K). The pipe is used to convey steam at a pressure of 700 kPa. The outside temperature of insulation is 24°C. If the pipe is 10 m long, determine the following, assuming the resistance to conductive heat transfer in steel pipe and convective resistance on the steam side are negligible:

a. The heat loss per hour.
b. The interface temperature of insulation.

***4.32** A 1-cm-thick steel pipe, 1 m long, with an internal diameter of 5 cm is covered with 4-cm-thick insulation. The inside wall temperature of the steel pipe is 100°C. The ambient temperature around the insulated pipe is 20°C. The convective heat-transfer coefficient on the outer insulated surface is 50 W/(m^2 K). Calculate the temperature at the steel insulation interface. The thermal conductivity of steel is 54 W/(m K), and the thermal conductivity of insulation is 0.04 W/(m K).

4.33 Calculate the overall heat-transfer coefficient of a steel pipe based on the inside area. The inside diameter of the pipe is 10 cm, and the pipe is 2 cm thick. The inside convective

* Indicates an advanced level of difficulty in solving.

heat-transfer coefficient is 350 W/(m^2 °C), the outside convective heat-transfer coefficient is 25 W/(m^2 °C), the thermal conductivity of the steel pipe is 15 W/(m °C). If the pipe is used to convey steam at a bulk temperature of 110°C and the outside ambient temperature is 20°C, determine the rate of heat transfer from the pipe.

***4.34** Saturated refrigerant (Freon, R-12) at −40°C flows through a copper tube of 20 mm inside diameter and wall thickness of 2 mm. The copper tube is covered with 40-mm-thick insulation ($k = 0.02$ W/[m K]). Determine the heat gain per meter of the pipe. The internal and external convective heat-transfer coefficients are 500 and 5 W/(m^2 K), respectively. The ambient air temperature is 25°C. Compare the amount of refrigerant vaporized per hour per meter length of pipe for insulated versus uninsulated pipe. The latent heat of the refrigerant at −40°C is 1390 kJ/kg.

4.35 To cool hot edible oil, an engineer has suggested that the oil be pumped through a pipe submerged in a nearby lake. The pipe (external diameter = 15 cm) will be located in a horizontal direction. The average outside surface temperature of the pipe will be 130°C. The surrounding water temperature may be assumed to be constant at 10°C. The pipe is 100 m long. Assume there is no movement of water.

a. Estimate the convective heat-transfer coefficient from the outside pipe surface into water.

b. Determine the rate of heat transfer from the pipe into water.

4.36 In a concurrent-flow tubular heat exchanger, a liquid food, flowing in the inner pipe, is heated from 20 to 40°C. In the outer pipe the heating medium (water) cools from 90 to 50°C. The overall heat-transfer coefficient based on the inside diameter is 2000 W/(m^2 °C). The inside diameter is 5 cm and length of the heat exchanger is 10 m. The average specific heat of water is 4.181 kJ/(kg °C). Calculate the mass flow rate of water in the outer pipe.

***4.37** A countercurrent heat exchanger is being used to heat a liquid food from 15 to 70°C. The heat exchanger has a 23-mm internal diameter and 10 m length with an overall

* Indicates an advanced level of difficulty in solving.

heat-transfer coefficient, based on the inside area, of 2000 W/(m^2 K). Water, the heating medium, enters the heat exchanger at 95°C, and leaves at 85°C. Determine the flow rates for product and water that will provide the conditions described. Use specific heats of 3.7 kJ/(kg K) for product and 4.18 kJ/(kg K) for water.

4.38 A 10-m-long countercurrent-flow heat exchanger is being used to heat a liquid food from 20 to 80°C. The heating medium is oil, which enters the heat exchanger at 150°C and exits at 60°C. The specific heat of the liquid food is 3.9 kJ/(kg K). The overall heat-transfer coefficient based on the inside area is 1000 W/(m^2 K). The inner diameter of the inside pipe is 7 cm.

a. Estimate the flow rate of the liquid food.

b. Determine the flow rate of the liquid food if the heat exchanger is operated in a concurrent-flow mode for the same conditions of temperatures at the inlet and exit from the heat exchanger.

4.39 Calculate the radiative heat gain in watts by a loaf of bread at a surface temperature of 100°C. The surrounding oven surface temperature is 1000°C. The total surface area of the bread is 0.15 m^2 and the emissivity of the bread surface is 0.80. Assume the oven is a blackbody radiator.

4.40 It is desired to predict the temperature after 30 min at the geometric center of a cylindrical can containing a model food. The dimensions of the can are 5 cm diameter and 3 cm height. The thermal conductivity of the food is 0.5 W/(m °C), specific heat = 3.9 kJ/(kg °C), and density = 950 kg/m^3. There is a negligible surface resistance to heat transfer. The surrounding medium temperature is 100°C and the uniform initial temperature of food is 20°C.

4.41 An 8-m^3 batch of oil with specific heat of 2 kJ/(kg K) and density of 850 kg/m^3 is being heated in a steam-jacketed, agitated vessel with 1.5 m^2 of heating surface. The convective heat-transfer coefficient on the oil side is 500 W/(m^2 K), and 10,000 W/(m^2 K) on the steam side. If the steam temperature is 130°C and the initial temperature is 20°C, estimate the oil temperature after 10 min.

4.42 Determine if a tubular heat exchanger can operate under the following conditions: Fluid A enters the heat exchanger at

120°C and exits at 40°C; fluid B enters the heat exchanger at 30°C and exits at 70°C. Calculate the log mean temperature difference.

4.43 Milk (c_p = 3.9 kJ/[kg K]) is cooled in a countercurrent flow heat exchanger at a rate of 1.5 kg/s from 70°C to 30°C. Cooling is done by using chilled water available at 5°C with a flow rate of 2 kg/s. The inside diameter of the inner pipe is 2 cm. The overall heat transfer coefficient is 500 W/(m^2 °C). Determine the length of the heat exchanger.

4.44 In a double-pipe heat exchanger, made of stainless steel (k = 15 W/[m °C]), the inside pipe has an inner diameter of 2 cm and an outside diameter of 2.5 cm. The outer shell has an inner diameter of 4 cm. The convective heat transfer coefficient on the inside surface of the inner pipe is 550 W/(m^2 °C), whereas on the outside surface of the inner pipe it is 900 W/(m^2 °C). Over continuous use of the heat exchanger, fouling (depositing of solids from the liquids on the pipe surfaces) causes additional resistance to heat transfer. It is determined that the resistance to heat transfer due to fouling on the inside surface of the inner pipe is 0.00038 m^2 °C/W, and on the outside surface of the inner pipe is 0.0002 m^2 °C/W. Calculate:

a. The total thermal resistance of the heat exchanger per unit length.

b. The overall heat transfer coefficients U_i and U_o based on the inside and outside area of the inner pipe, respectively.

4.45 Water at 5°C is being used to cool apples from an initial temperature of 20° to 8°C. The water flow over the surface of the apple creates a convective heat-transfer coefficient of 10 W/(m^2 K). Assume the apple can be described as a sphere with an 8-cm diameter and the geometric center is to be reduced to 8°C. The apple properties include thermal conductivity of 0.4 W/(m K), specific heat of 3.8 kJ/(kg K), and density of 960 kg/m^3. Determine the time that the apples must be exposed to the water.

4.46 A liquid food with density of 1025 kg/m^3 and specific heat of 3.77 kJ/(kg K) is being heated in a can with 8.5 cm diameter and 10.5 cm height. The heating will occur in a retort with temperature at 115°C and convective heat-transfer coefficients of 50 W/(m^2 K) on the inside of the can and

5000 W/(m^2 K) on the outside surface. Determine the product temperature after 10 minutes if the initial temperature is 70°C. Assume perfect mixing in the can.

4.47 Create a spreadsheet for Example 4.21. Keeping all conditions the same as given in the example, determine the exit temperatures of hot water and apple juice if the number of plates used in the heat exchanger are 21 or 31.

***4.48** A conduction-cooling food product with density of 1000 kg/m^3, specific heat of 4 kJ/(kg K), and thermal conductivity of 0.4 W/(m K) has been heated to 80°C. The cooling of the product in a 10 cm high and 8 cm diameter can is accomplished using cold water with a convective heat-transfer coefficient of 10 W/(m^2 K) on the can surface. Determine the water temperature required to reduce the product temperature at geometric center to 50°C in 7 h. Neglect conductive heat resistance through the can wall.

4.49 Cooked mashed potato is cooled on trays in a chilling unit with refrigerated air at 2°C blown over the product surface at high velocity. The depth of product is 30 mm and the initial temperature is 95°C. The product has a thermal conductivity of 0.37 W/(m K), specific heat of 3.7 kJ/(kg K), and density of 1000 kg/m^3. Assuming negligible resistance to heat transfer at the surface, calculate product temperature at the center after 30 minutes.

4.50 Program Example 4.9 on a spreadsheet. Determine the interfacial temperatures if the following thickness of insulation are used:

a. 2 cm
b. 4 cm
c. 6 cm
d. 8 cm
e. 10 cm

4.51 A liquid food at a flow rate of 0.3 kg/s enters a countercurrent flow double-pipe heat exchanger at 22°C. In the annular section, hot water at 80°C enters at a flow rate of 1.2 kg/s. The average specific heat of water is 4.18 kJ/(kg °C). The average overall heat transfer coefficient based on the inside area

---

* Indicates an advanced level of difficulty in solving.

is 500 W/(m^2 °C). The diameter of the inner pipe is 7 cm, and length is 10 m. Assume steady state conditions. The specific heat of liquid food is assumed to be 4.1 kJ/[kg °C]. Calculate the exit temperature of liquid food and water.

4.52 A pure copper sphere of radius 1 cm is dropped into an agitated oil bath that has a uniform temperature of 130°C. The initial temperature of the copper sphere is 20°C. Using a spreadsheet predict the internal temperature of the sphere at 5-min intervals until it reaches 128°C for three different convective heat-transfer coefficients: 5, 10, and 100 W/(m^2 °C), respectively. Plot your results as temperature versus time.

4.53 Heated water with a bulk temperature of 90°C is being pumped at a rate of 0.1 kg/s in a metal pipe placed horizontally in ambient air. The pipe has an internal diameter of 2.5 cm and it is 1 cm thick. The inside pipe surface temperature is 85°C. The outside surface of the pipe is at 80°C and exposed to the air. The bulk temperature of the air is 20°C.

a. Determine the convective heat-transfer coefficient for water inside the pipe.

b. Determine the convective heat-transfer coefficient for air outside the pipe.

c. It is desired to double the convective heat-transfer coefficient inside the pipe. What operating conditions should be changed? By how much?

4.54 A solid food is being cooled in a cylindrical can of dimensions 12 cm diameter and 3 cm thickness. The cooling medium is cold water at 2°C. The initial temperature of the solid food is 95°C. The convective heat-transfer coefficient is 200 W/(m^2 °C).

a. Determine the temperature at the geometric center after 3 h. The thermal properties of the solid food are $k = 0.36$ W/(m °C), density of 950 kg/m^3, and specific heat of 3.9 kJ/(kg °C).

b. Is it reasonable to assume the cylindrical can to be an infinite cylinder (or an infinite slab)? Why?

4.55 A three-layered composite pipe with an inside diameter of 1 cm has an internal surface temperature of 120°C. The first layer, from the inside to the outside, is 2 cm thick with a thermal conductivity of 15 W/(m °C), the second layer is 3 cm thick with a thermal conductivity of 0.04 W/(m °C), and the third layer is 1 cm thick and has a thermal conductivity of 164 W/(m °C). The outside surface temperature of the composite pipe is 60°C.

a. Determine the rate of heat transfer through the pipe under steady-state conditions.

b. Can you suggest an approach that will allow you to quickly make an estimate for this problem?

4.56 It is known that raw eggs will become hard when heated to 72°C. To manufacture diced eggs, trays of liquid egg are exposed to steam for cooking.

a. How long will it take to cook the eggs given the following conditions? The tray dimension is 30 cm long, 30 cm wide, and 2 cm deep. The liquid egg inside the tray is 1 cm deep. The thermal conductivity of liquid egg is 0.45 W/(m °C); density is 800 kg/m^3, specific heat is 3.8 kJ/(kg °C); surface convective heat transfer coefficient is 5000 W/(m^2 °C); and the initial temperature of liquid egg is 2°C. Steam is available at 169.06 kPa. Ignore resistance to heat transfer caused by the metal tray.

b. What rate of steam flow per tray of liquid egg must be maintained to accomplish this? The latent heat of vaporization at 169.06 kPa is 2216.5 kJ/kg.

4.57 Determine the time required for the center temperature of a cube to reach 80°C. The cube has a volume of 125 cm^3. The thermal conductivity of the material is 0.4 W/(m °C); density is 950 kg/m^3; and specific heat is 3.4 kJ/(kg K). The initial temperature is 20°C. The surrounding temperature is 90°C. The cube is immersed in a fluid that results in a negligible surface resistance to heat transfer.

4.58 A tubular heat exchanger is being used for heating a liquid food from 30° to 70°C. The temperature of the heating medium decreases from 90° to 60°C.

a. Is the flow configuration in the heat exchanger countercurrent or concurrent flow?

b. Determine the log mean temperature difference.

c. If the heat transfer area is 20 m^2 and the overall heat transfer coefficient is 100 W/(m^2 °C), determine the rate of heat transfer from the heating medium to the liquid food.

d. What is the flow rate of the liquid food if the specific heat of the liquid is 3.9 kJ/(kg °C)? Assume no heat loss to the surroundings.

4.59 What is the flow rate of water in a heat exchanger if it enters the heat exchanger at 20°C and exits at 85°C? The heating medium is oil, where oil enters at 120°C and leaves at 75°C. The overall heat-transfer coefficient is 5 W/(m^2 °C). The area of the heat exchanger is 30 m^2.

4.60 A thermocouple is a small temperature sensor used in measuring temperature in foods. The thermocouple junction, that senses the temperature, may be approximated as a sphere. Consider a situation where a thermocouple is being used to measure temperature of heated air in an oven. The convective heat-transfer coefficient is 400 W/(m^2 K). The properties of the thermocouple junction are k = 25 W/(m °C), cp = 450 J /(kg K), and p = 8000 kg/m^3. The diameter of the junction, considered as a sphere, is 0.0007 m. If the junction is initially at 25°C, and it is placed in the oven where the estimated heated air temperature is 200°C, how long will it take for the junction to reach 199°C?

4.61 In an ohmic heater, the inside diameter of the pipe is 0.07 m and it is 2 m long. The applied voltage is 15,000 V. A liquid food is being pumped through the heater at 0.2 kg/s with an inlet temperature of 30°C. The overall heat transfer coefficient based on the inside pipe area is 200 W/m^2 °C. The specific heat of the liquid food is 4000 J/kg °C. The surrounding temperature of the air is 25°C. Assume that the properties of the liquid food are similar to 0.1 M Sodium Phosphate solution. Determine the exit temperature of the liquid food.

4.62 A liquid food at 75°C is being conveyed in a steel pipe (k = 45 W/[m °C]). The pipe has an internal diameter of 2.5 cm and it is 1 cm thick. The overall heat-transfer coefficient based on internal diameter is 40 W/(m^2 K). The internal convective heat-transfer coefficient is 50 W/(m^2 K). Calculate the external convective heat-transfer coefficient.

4.63 Set up Example 4.16 on a spreadsheet. Determine the heat loss from 1 m length of the pipe if the inside diameter of the pipe is

a. 2.5 cm
b. 3.5 cm
c. 4.5 cm
d. 5.5 cm

***4.64** A plot of tomato juice density versus temperature was scanned and digitized from Choi and Okos (1983). Juice density versus solids content for several temperatures are given in the following table.

Solids	$T = 30°C$	$T = 40°C$	$T = 50°C$	$T = 60°C$	$T = 70°C$	$T = 80°C$
0%	997	998	984	985	979	972
4.8%	1018	1018	1012	1006	1003	997
8.3%	1032	1032	1026	1026	1020	1017
13.9%	1070	1067	1064	1061	1058	1048
21.5%	1107	1108	1102	1102	1093	1086
40.0%	1190	1191	1188	1185	1179	1176
60.0%	1290	1294	1288	1289	1286	1276
80.0%	1387	1391	1385	1382	1379	1376

Create a MATLAB® script that plots the values of density as a function of percent solids at a temperature of 40°C. Use the Basic Fitting option under the Tools menu of the Figure window to fit the data with an appropriate polynomial. Turn in a copy of your script, plot, and the equation that you determine.

***4.65** A plot of tomato juice density versus temperature was scanned and digitized by Choi and Okos (1983). Values of juice density versus solids content for several temperatures are shown in a table in Problem 4.64. Write a MATLAB® script to evaluate and plot the model for tomato juice density developed by Choi and Okos.

$$\rho = \rho_s X_s + \rho_w X_w$$
$$\rho_w = 9.9989 \times 10^2 - 6.0334 \times 10^{-2} T - 3.6710 \times 10^{-3} T^2$$
$$\rho_s = 1.4693 \times 10^3 + 5.4667 \times 10^{-1} T - 6.9646 \times 10^{-3} T^2$$

ρ = density (kg/m^3)
ρ_w = density of water (kg/m^3)
ρ_s = density of solids (kg/m^3)
T = temperature of juice (°C)
X_s = percent solids in juice (%)
X_w = percent water in juice (%)

Plot this model over the preceding experimental data for 40°C.

* Indicates an advanced level of difficulty in solving.

*4.66 Telis-Romero et al. (1998) presented data for the specific heat of orange juice as a function of temperature and percent water. A portion of their data was digitized and is given in the following table.

	$T = 8°C$	$T = 18°C$	$T = 27°C$	$T = 47°C$	$T = 62°C$
X_w (w/w)	c_p (kJ/kg °C)	c_p (kJ/kg °C)	c_p (kJ/kg °C)	c_p (kJ/kg °C)	c_p (kJ/kg °C)
0.34	2.32	2.35	2.38	2.43	2.45
0.40	2.49	2.51	2.53	2.59	2.61
0.44	2.59	2.62	2.64	2.68	2.72
0.50	2.74	2.78	2.80	2.85	2.88
0.55	2.88	2.91	2.93	2.98	3.01
0.59	2.99	3.01	3.03	3.08	3.12
0.63	3.10	3.12	3.14	3.19	3.23
0.69	3.26	3.28	3.30	3.36	3.39
0.73	3.37	3.39	3.41	3.46	3.49

Notice that specific heat is a function of temperature as well as solids content for all juices.

Two general-purpose empirical equations are often used to estimate the specific heat for plant material and their juices. The first equation is known as Siebel's correlation equation, and is used to estimate the specific heat of "fat-free fruits and vegetables, purees, and concentrates of plant origin:"

$$c_p = 3.349(X_w) + 0.8374$$

The second equation is given from the ASHRAE Handbook—Fundamentals (2005):

$$c_p = 4.187(0.6X_w + 0.4)$$

Using MATLAB®, create a plot of c_p vs moisture content using data given in the table for all temperatures and compare with calculated values using the two empirical equations.

*4.67 The solution to temperature in an infinite slab with finite internal and surface resistance to heat transfer is given by Bergman et al. (2011) as

$$\frac{T - T_a}{T_i - T_a} = \sum_{n=1}^{\infty} C_n \exp(-\beta_n^2 N_{Fo})\cos(\beta_n x/d_c)$$

* Indicates an advanced level of difficulty in solving.

$$C_n = \frac{4\sin(\beta_n)}{2\beta_n + \sin(2\beta_n)}$$

$$\beta_n \tan \beta_n = N_{Bi}$$

They give the first four roots of the transcendental equation for β for a range of Biot numbers. The first four roots for $N_{Bi} = 0.5$ are $\beta_1 = 0.6533$, $\beta_2 = 3.2923$, $\beta_3 = 6.3616$, and $\beta_4 = 9.4775$. Write a MATLAB® script to evaluate the temperature for a slab with

$$d_c = 0.055 \text{ (m)}$$
$$k = 0.34 \text{ (W/m °C)}$$
$$c_p = 3500 \text{ (J/kg °C)}$$
$$\rho = 900 \text{ (kg/m}^3\text{)}$$
$$N_{Bi} = 0.5$$
$$T_a = 100 \text{ (°C)}$$
$$T_i = 35 \text{ (°C)}$$

Plot the temperatures $T(x,t)$ over the range of x from 0 to 0.055 m for times of 20, 40, and 60 min.

***4.68** Write a MATLAB® script to use the MATLAB® function *pdepe* to evaluate the temperature for an infinite slab with

$$d_c = 0.055 \text{ (m)}$$
$$k = 0.34 \text{ (W/m °C)}$$
$$c_p = 3500 \text{ (J/kg °C)}$$
$$\rho = 900 \text{ (kg/m}^3\text{)}$$
$$N_{Bi} = 0.5$$
$$T_a = 100 \text{ (°C)}$$
$$T_i = 35 \text{ (°C)}$$

Plot the temperatures $T(x,t)$ over the range of x from 0 to 0.055 m for times of 20, 40, and 60 min.

LIST OF SYMBOLS

A	area (m^2)
a	coefficient in Equation (4.70)
α	thermal diffusivity (m^2/s)

* Indicates an advanced level of difficulty in solving.

α'	attenuation factor (m^{-1})
b	gap between two adjacent plates (m)
β	coefficient of volumetric expansion (K^{-1})
C_F	cleaning factor, dimensionless
C_H	heat capacity rate (kJ/[s °C])
C_{min}	minimum heat capacity rate (kJ/[s °C])
C^*	heat capacity rate ratio, dimensionless
c_p	specific heat at constant pressure (kJ/[kg °C])
c_v	specific heat at constant volume (kJ/[kg °C])
χ	reflectivity, dimensionless
D	diameter (m)
d_c	characteristic dimension (m)
D_e	equivalent diameter (m)
d	penetration depth (m)
E	electrical field strength (V/cm)
E_V	voltage (V)
ε	emissivity, dimensionless
ε_E	heat exchanger effectiveness, dimensionless
ε'	relative dielectric constant, dimensionless
ε''	relative dielectric loss constant, dimensionless
F	shape factor, dimensionless
f_h	heating rate factor (s)
f	friction factor, dimensionless
f'	frequency (Hz)
g	acceleration due to gravity (m/s^2)
h	convective heat transfer coefficient (W/[m^2 K])
I	electric current (A)
J_0	Bessel function of zero order
J_1	Bessel function of first order
j_c	temperature lag factor at center
j_m	temperature lag factor, mean
K	coefficient in Equation (4.185)
k	thermal conductivity (W/[m K])
κ_E	electrical conductance (siemens)
L	length (m)
l	thickness of fluid layer (m)
λ	wavelength (m)
λ_n	eigenvalue roots
M	mass concentration, percent
m	mass (kg); coefficient in Equations (4.59) and (4.70)
m''	coefficient in Equation (4.184)
$\dot{m}$	mass flow rate (kg/s)

μ	viscosity (Pa s)
N	number of plates
N_{Bi}	Biot number, dimensionless
N_{Fo}	Fourier number, dimensionless
N_{Gr}	Grashoff number, dimensionless
N_{Nu}	Nusselt number, dimensionless
N_{Pr}	Prandtl number, dimensionless
N_{Re}	Reynolds number, dimensionless
N_{Ra}	Raleigh number, dimensionless
n	coefficient in Equation (4.59)
ν	kinematic viscosity (m^2/s)
P	power at the penetration depth (W)
P_D	power dissipation (W/cm^3)
P_o	incident power (W/cm^3)
ϕ	absorptivity, dimensionless
Φ	function
ψ	transmissivity, dimensionless
Q	heat gained or lost (kJ)
q	rate of heat transfer (W)
q'''	rate of heat generation (W/m^3)
R_E	electrical resistance (Ω)
R_f	fouling resistance, [(m^2 °C]/W)
R_t	thermal resistance (°C/W)
r	radius or variable distance in radial direction (m)
r_c	critical radius
ρ	density (kg/m^3)
σ	Stefan–Boltzmann constant (5.669×10^{-8}W/[m^2 K^4])
σ_E	electrical conductivity (S/m)
σ_o	electrical conductivity at 0°C (S/m)
T	temperature (°C)
t	time (s)
T_e	exit temperature (°C)
T_A	absolute temperature (K) or pseudo initial temperature for Ball's method (°C)
T_a	temperature of surrounding medium (°C)
T_f	film temperature (°C)
T_i	initial or inlet temperature (°C)
T_p	plate surface temperature (°C)
T_s	surface temperature (°C)
T_∞	fluid temperature far away from solid surface (°C)
$\tan\delta$	loss tangent, dimensionless
U	overall heat transfer coefficient (W/[m^2 °C])

$\overline{u}$	velocity (m/s)
V	volume (m^3)
w	width of a plate (m)
X	mass fraction, dimensionless
x	variable distance in x direction (m)
Y	volume fraction, dimensionless
Z	depth (m)
z	space coordinate
ξ	factor to account for shape and emissivity, dimensionless

Subscripts: a, ash; b, bulk; c, channel; ci, inside clean surface; co, outside clean surface; f, fat; fi, inside fouled surface; fo, outside fouled surface; h, carbohydrate; i, inside surface; lm, log mean; m, moisture; o, outside surface; p, protein; r, radial direction; s solid; x, x-direction; w, at wall (or water); H, hot stream; C, cold stream; P, product stream.

BIBLIOGRAPHY

American Society of Heating, Refrigerating and Air Conditioning Engineers, Inc., 2009. ASHRAE Handbook—Fundamentals. ASHRAE, Atlanta, Georgia.

Bergman, T.L., Lavine, A.S., Incropera, F.P., DeWitt, D.P., 2011. Fundamentals of Heat and Mass Transfer, seventh ed. John Wiley and Sons, inc., New York.

Brennan, J.G., Butters, J.R., Cowell, N.D., Lilly, A.E.V., 1990. Food Engineering Operations, third ed. Elsevier Science Publishing Co., New York.

Cengel, Y.A., Boles, M.A., 2010. Thermodynamics. An Engineering Approach, seventh ed. McGraw Hill, Boston.

Charm, S.E., 1978. The Fundamentals of Food Engineering, third ed. AVI Publ. Co., Westport, Connecticut.

Choi, Y., Okos, M.R., 1986. Effects of temperature and composition on the thermal properties of food. In: Le Maguer, M., Jelen, P. (Eds.), Food Engineering and Process Applications, Vol. 1. "Transport Phenomena". Elsevier Applied Science Publishers, London, pp. 93−101.

Choi, Y., Okos, M.R., 1983. The thermal properties of tomato juice concentrates. Trans. ASAE 26 (1), 305−315.

Copson, D.A., 1975. Microwave Heating. AVI Publ. Co., Westport, Connecticut.

Coulsen, J.M., Richardson, J.F., 1999. Chemical Engineering, Vol. 1. Pergamon Press, Oxford.

Das, S.K., 2005. Process Heat Transfer. Alpha Science International Ltd., Harrow, UK.

Decareau, R.V., 1992. Microwave Foods: New Product Development. Food and Nutrition Press, Trumbull, Connecticut.

Decareau, R.V., Peterson, R.A., 1986. Microwave Processing and Engineering. VCH Publ., Deerfield Beach, Florida.

Dickerson Jr., R.W., 1969. Thermal properties of foods. In: Tressler, D.K., Van Arsdel, W.B., Copley, M.J. (Eds.), The Freezing Preservation of Foods, fourth ed., Vol. 2. AVI Publ. Co., Westport, Connecticut, pp. 26–51.

Giese, J., 1992. Advances in microwave food processing. Food Tech. 46 (9), 118–123.

Green, D.W., Perry, R.H., 2007. Perry's Chemical Engineer's Handbook, eighth ed. McGraw-Hill, New York.

Heldman, D.R., Lund, D.B., 2007. Handbook of Food Engineering. CRC Press, Taylor & Francis Group, Boca Raton, Florida.

Heldman, D.R., Singh, R.P., 1981. Food Process Engineering, second ed. AVI Publ. Co., Westport, Connecticut.

Holman, J.P., 2009. Heat Transfer, Tenth ed. McGraw-Hill, New York.

Hottel, H.C., 1930. Radiant heat transmission. Mech. Eng. 52 (7), 700.

IFT, 1989. Microwave food processing. A scientific status summary by the IFT expert panel on food safety and nutrition. Food Tech. 43 (1), 117–126.

Kakac, S., Liu, H., 1998. Heat Exchangers. Selection, Rating and Thermal Design. CRC Press, Boca Raton, Florida.

Kreith, F., 1973. Principles of Heat Transfer. IEP-A Dun-Donnelley Publisher, New York.

Lin, Z., Cleland, A.C., Sellarach, G.F., Cleland, D.J., 1996. A simple method for prediction of chilling times: Extension to three-dimensional irregular shapes. Int. J. Refrigeration 19, 107–114, Erratum, Int. J. Refrigeration. 23, 168.

Mertens, B., Knorr, D., 1992. Developments of nonthermal processes for food preservation. Food Tech. 46 (5), 124–133.

Mudgett, R.E., 1986. Microwave properties and heating characteristics of foods. Food Tech. 40 (6), 84–93.

Myers, G.E., 1971. Analytical Methods in Conduction Heat Transfer. McGraw-Hill, New York.

Ohlsson, T., 1983. Fundamentals of microwave cooking. Microw. World 4 (2), 4.

Palaniappan, S., Sastry, S.K., 1991. Electrical conductivity of selected solid foods during ohmic heating. J. Food Proc. Engr. 14, 221–236.

Pflug, I.J., Blaisdell, J.L., Kopelman, J., 1965. Developing temperature-time curves for objects that can be approximated by a sphere, infinite plate or infinite cylinder. ASHRAE Trans. 71 (1), 238–248.

Pham, Q.T., 2001. Prediction of cooling/freezing time and heat loads. In: Sun, D.W. (Ed.), Advances in Food Refrigeration. Leatherhead International, Leatherhead, Surrey, UK.

Rotstein, E., Singh, R.P., Valentas, K.J., 1997. Handbook of Food Engineering Practice. CRC Press, Boca Raton, Florida.

Schiffman, R.F., 1986. Food product development for microwave processing. Food Tech. 40 (6), 94–98.

Schneider, P.J., 1963. Temperature Response Charts. Wiley, New York.

Siebel, J.E., 1892. Specific heat of various products. Ice Refrigeration 2, 256.

Singh, R.P., 1994. Food Properties Database. RAR Press, Davis, California.

Sweat, V.E., 1974. Experimental values of thermal conductivity of selected fruits and vegetables. J. Food Sci. 39, 1080.

Sweat, V.E., 1975. Modeling thermal conductivity of meats. Trans. Am. Soc. Agric. Eng. 18 (1), 564–565, 567, 568.

Sweat, V.E., 1986. Thermal properties of foods. In: Rao, M.A., Rizvi, S.S.H. (Eds.), Engineering Properties of Foods. Marcel Dekker Inc., New York, pp. 49–87.

Telis-Romero, J., Telis, V.R.N., Gabas, A.L., Yamashita, F., 1998. Thermophysical properties of Brazilian orange juice as affected by temperature and water content. J. Food Engr. 38, 27–40.

von Hippel, A.R., 1954. Dielectrics and Waves. MIT Press, Cambridge, Massachusetts.

Watt, B.K., Merrill, A.L., 1975. Composition of Foods, Raw, Processed, Prepared. Agriculture Handbook, Number 8. United States Department of Agriculture, Washington, D.C.

Whitaker, S., 1977. Fundamental Principles of Heat Transfer. Krieger Publishing Co., Melbourne, Florida.